Electromagnons & quantum critical behavior

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Electromagnons:

Spin-wave excitations carrying an electric dipole (electro-active spin-waves)

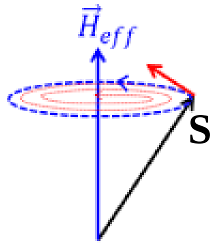
Baryakhtar & Chupis, Sov. Phys. Solid State 10, 2818 (69)
Pimenov et al., Nature Phys. 2, 97 (06)

Conversely, phonons can also carry a magnetic moment (magneto-active phonons)

Dzyaloshinskii & Mills, Phil. Mag. 89, 2079 (09)
Chaix et al. PRL 110, 157208 (13)
Juraschek et al. Phys. Rev. Materials 1, 014401 (17)

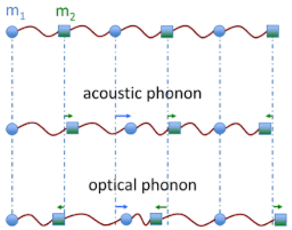
Spins

$$\dot{\mathbf{S}} = -\mathbf{S} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{S} \times \dot{\mathbf{S}}$$



Phonons

$$\ddot{\mathbf{u}} + \gamma \dot{\mathbf{u}} = \omega_0^2 \mathbf{u} + \beta u^2 \mathbf{u}$$



Intertwined dynamics $\longrightarrow$ specific signatures in the quantum critical behavior

Narayan, Cano, Balatsky & Spaldin, Nature Mater. 18, 223 (19)

Phase transitions: classical picture

Partition function:

$$
\begin{aligned}
Z &= \int e^{-\beta \sum_n \overbrace{H(\mathbf{p}_n, \mathbf{x}_n)}^{H = T(\mathbf{p}_n) + V(\mathbf{x}_n)}} \Pi_n dp_n dx_n \\
&= Z_{\mathrm{kin}} \int e^{-\beta \sum_n V(\mathbf{x}_n)} \Pi_n dx_n \\
&= Z_{\mathrm{kin}} \int e^{-\beta \sum_n V(\eta;\boldsymbol{\xi}_n)} d\eta \Pi_n d\xi_n \\
&= Z_{\mathrm{kin}} \int e^{-\beta \sum_n V_{\mathrm{eff}}(\beta;\eta)} d\eta
\end{aligned}
$$

Phase transitions: classical picture

Partition function:

$$H = T(\mathbf{p}_n) + V(\mathbf{x}_n)$$

$$Z = \int e^{-\beta \sum_n \overbrace{H(\mathbf{p}_n, \mathbf{x}_n)}} \Pi_n dp_n dx_n$$

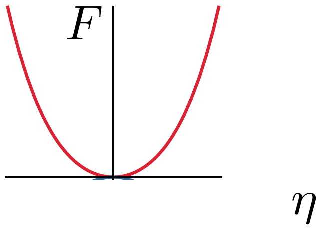
$$= Z_{\text{kin}} \int e^{-\beta \sum_n V(\mathbf{x}_n)} \Pi_n dx_n$$

$$= Z_{\text{kin}} \int e^{-\beta \sum_n V(\eta; \boldsymbol{\xi}_n)} d\eta \Pi_n d\xi_n$$

$$= Z_{\text{kin}} \int e^{-\beta \sum_n V_{\text{eff}}(\beta; \eta)} d\eta$$

Free energy: $F = -k_B T \ln Z = F_{\text{kin}} + F_{\text{Landau}}(T; \eta)$

$$F_{\text{Landau}}(T; \eta) = F_0 + \frac{1}{2} a'(T - T_c)\eta^2 + \frac{b}{4}\eta^4$$



Quantum phase transitions

Rechester, Sov. Phys. JETP 33, 423 (71)
Khmel'nitskii & Shneerson, Sov. Phys. Solid State 13, 687 (71)

Position $\hat{\mathbf{x}}$ and momentum $\hat{\mathbf{p}}$ operators do not commute: $\hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$

$$Z = \int e^{-\beta \sum_n \overbrace{H(\mathbf{p}_n, \mathbf{x}_n)}^{H = T(\mathbf{p}_n) + V(\mathbf{x}_n)}} \Pi_n dp_n dx_n$$

$$\neq Z_{\rm kin} \int e^{-\beta \sum_n V(\mathbf{x}_n)} \Pi_n dx_n$$

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Dynamics plays a role in the thermodynamics

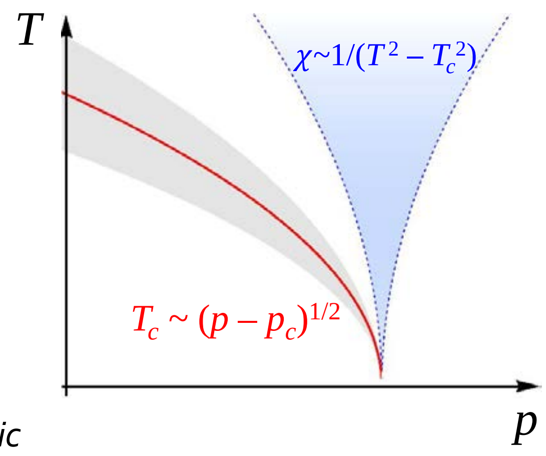
$$m\ddot{\eta} = (a + b\eta^2 + c\nabla^2)\eta$$

$$F = F_0 + \sum_{\mathbf{k}} \frac{1}{2}\hbar\omega(\mathbf{k}, \eta_0) + k_B T \sum_{\mathbf{k}} \ln[1 - e^{-\hbar\omega(\mathbf{k},\eta_0)/k_B T}]$$

$$= F_0 + \frac{1}{2} a'(T^2 - T_c^2)\eta_0^2 + \ldots$$

$$a(T = 0) = a_0 + a_{\text{quantum fluctuations}} > a_0$$

Quantum fluctuations stabilize the symmetric phase → *quantum paraelectric*



Quantum phase transitions

Cano & Levanyuk, Ferroelectrics 283, 3 (03)
Cano & Levanyuk PRB 70, 064104 (04)

Coupled order parameters

Long-range interactions change the form of the phase diagram and the asymptotic behaviors

Uniaxial ferroelectrics

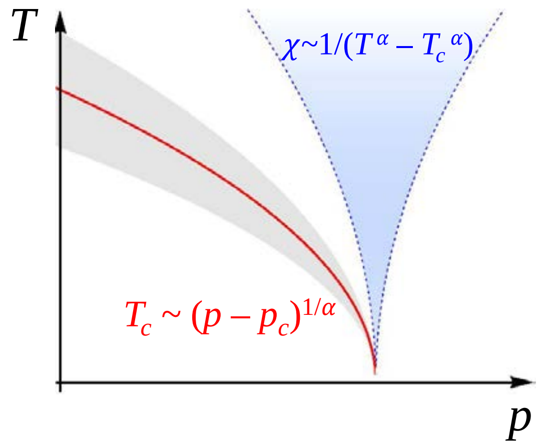
$$U = \frac{a}{2}\eta^2 + \frac{b}{4}\eta^4 + \frac{c}{2}(\nabla\eta)^2 \underbrace{+g\eta(\partial_z\phi)}_{-P_zE_z} + \frac{\varepsilon}{2}(\nabla\phi)^2$$

$$\alpha = 3$$

Pseudo-proper ferroelastics

$$U = \frac{a}{2}\eta^2 + \frac{b}{4}\eta^4 + \frac{c}{2}(\nabla\eta)^2 + g\eta u_{xy} + \frac{\lambda}{2}u_{ii}^2 + \mu u_{ik}^2$$

$$\alpha = 5/2$$



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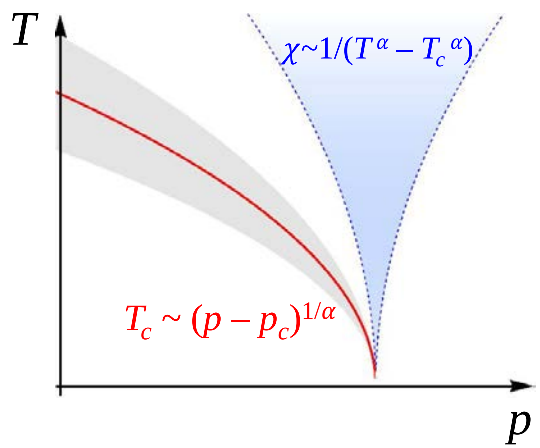

Pseudo-proper ferroelastics

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$$\alpha = 5/2$$

Dynamics

$$\tau \sim \xi^z \sim |T - T_c|^{-z\nu}$$

displacive phase transition

$$m\ddot{\eta} + \gamma\dot{\eta} = (a + b\eta^2 + c\nabla^2)\eta \quad (\gamma\dot{\eta} \to 0)$$

$$S = \int d^d\mathbf{k}d\omega(\tilde{m}\omega^2 + \xi^{-2} + k^2)|\eta_{\mathbf{k},\omega}|^2 + \dots$$

$$\alpha = 2$$

vs

order-disorder

$$m\ddot{\eta} + \gamma\dot{\eta} = (a + b\eta^2 + c\nabla^2)\eta \quad (m\ddot{\eta} \to 0)$$

$$S = \int d^d\mathbf{k}d\omega\left(\tilde{\gamma}\frac{|\omega|}{k^{z-2}} + \xi^{-2} + k^2\right)|\eta_{\mathbf{k},\omega}|^2 + \dots$$

$$\alpha = (z+1)/2$$

Quantum phase transitions

Coupled order parameters with different dynamics

$$\begin{cases} \phi & \text{ferroelectric order } (z = 1) \\ \psi & \text{magnetic order } (z = 2, 3) \end{cases}$$

$$S_{\mathrm{tot}} = S_\phi + S_\psi + S_{\mathrm{int}}$$

$$S_\phi = \int d^d\mathbf{k} d\omega (\tilde{m}\omega^2 + \xi^{-2} + k^2)|\boldsymbol{\phi}_{\mathbf{k},\omega}|^2 + \dots$$

$$S_\psi = \int d^d\mathbf{k} d\omega \left(\tilde{\gamma}\frac{|\omega|}{k^{z-2}} + \xi^{-2} + k^2\right)|\boldsymbol{\psi}_{\mathbf{k},\omega}|^2 + \dots$$

Quantum phase transitions

Coupled order parameters with different dynamics

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$$S_\phi = \int d^d\mathbf{k}d\omega(\tilde{m}\omega^2 + \xi^{-2} + k^2)|\boldsymbol{\phi}_{\mathbf{k},\omega}|^2 + \ldots$$

$$S_\psi = \int d^d\mathbf{k}d\omega\Big(\tilde{\gamma}\frac{|\omega|}{k^{z-2}} + \xi^{-2} + k^2\Big)|\boldsymbol{\psi}_{\mathbf{k},\omega}|^2 + \ldots$$

crossover

$$\chi_\phi^{-1} \sim T^2 \to T^{3/2}$$

Interactions

$$S_{\rm int}^0 = g_0 \int d^d\mathbf{r}d\tau\ |\boldsymbol{\phi}|^2|\boldsymbol{\psi}|^2$$

Electromagnons

$$S_{\rm int}^1 = g_1 \int d^d\mathbf{r}d\tau\ \boldsymbol{\phi}\cdot[\boldsymbol{\psi}(\nabla\cdot\boldsymbol{\psi}) - (\boldsymbol{\psi}\cdot\nabla)\boldsymbol{\psi}]$$

$$S_{\rm int}^2 = g_2 \int d^d\mathbf{r}d\tau\ \boldsymbol{\psi}\cdot(\boldsymbol{\phi}\times\partial_\tau\boldsymbol{\phi})$$

subdominant corrections

$$\chi_\phi^{-1} \sim T^{2z} = T^4$$

$$\chi_\phi^{-1} \sim T^{3-1/z} = T^{5/2}$$

Quantum phase transitions

Materials

$EuTiO_3$ G-type AMF & quantum paraelectric

PE-AFM **FE - FM**

crossover

$$\chi_\phi^{-1} \sim T^2 \rightarrow T^{3/2}$$

subdominant corrections

$$\chi_\phi^{-1} \sim T^{2z} = T^4$$

$$\chi_\phi^{-1} \sim T^{3-1/z} = T^{5/2}$$

Other candidates: $(Sr,Mn)TiO_3$, $(Tb,Y)MnO_3$

Conclusions

- Multiferroic quantum criticality signals the different dynamics of the coupled orders and their interplay

- Electromagnon couplings are fully relevant in the quantum regime

Narayan, Cano, Balatsky & Spaldin, Nature Mater. 18, 223 (19)