# Electric control of non-collinear magnets


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Zernike Institute

for Advanced Materials



Electromagnon workshop

Orsay

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# Collaborators

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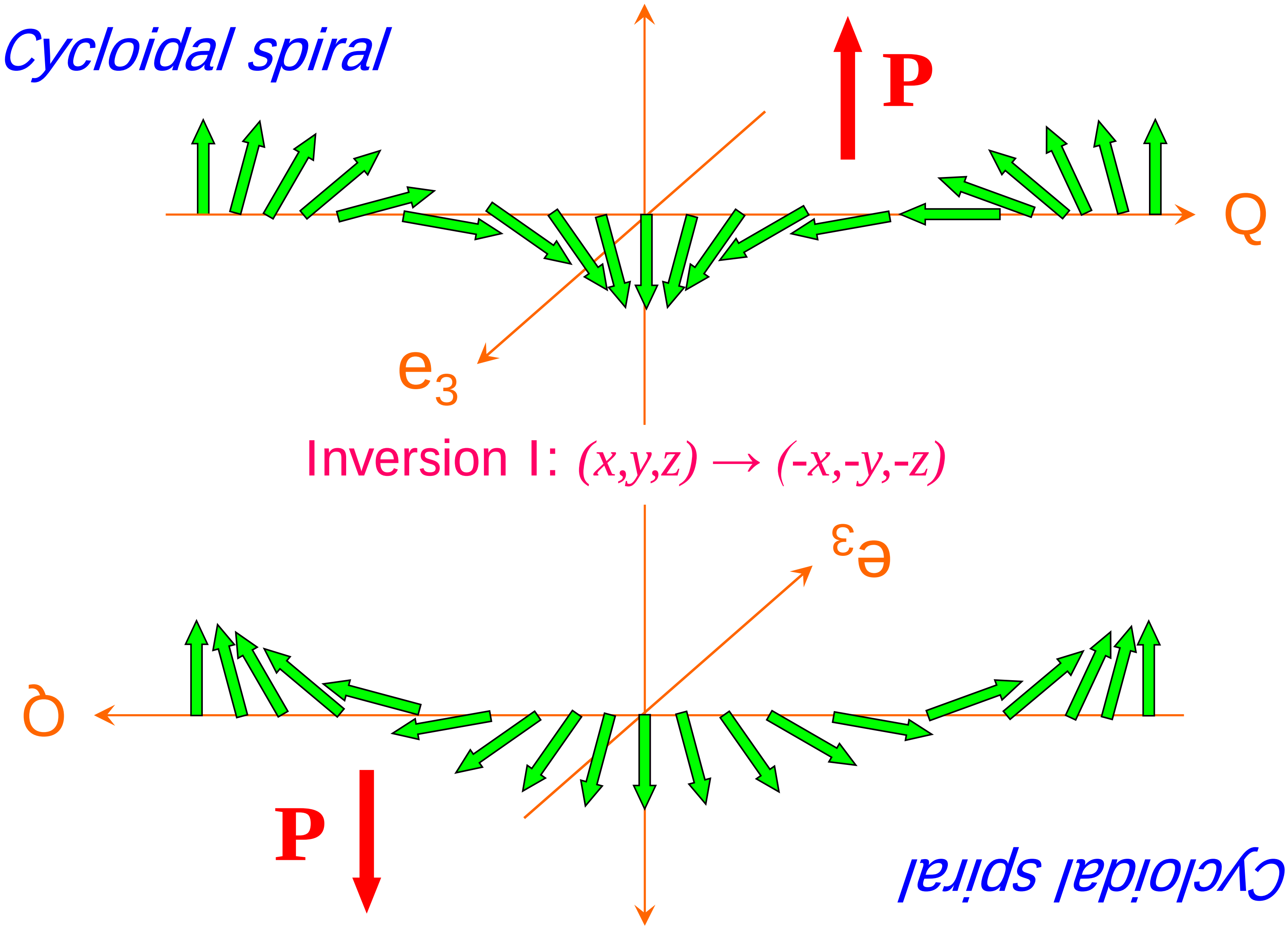
# Outline

- Symmetry of (electro-)magnons in spiral magnets
- Electric field control of polarization rotation
- Electromagnon and skyrmion mass
- Electrically excited dynamics of topological magnetic defects

# Electromagnons in $RMnO_3$

# Breaking of inversion symmetry by spin ordering



# Directional dichroism

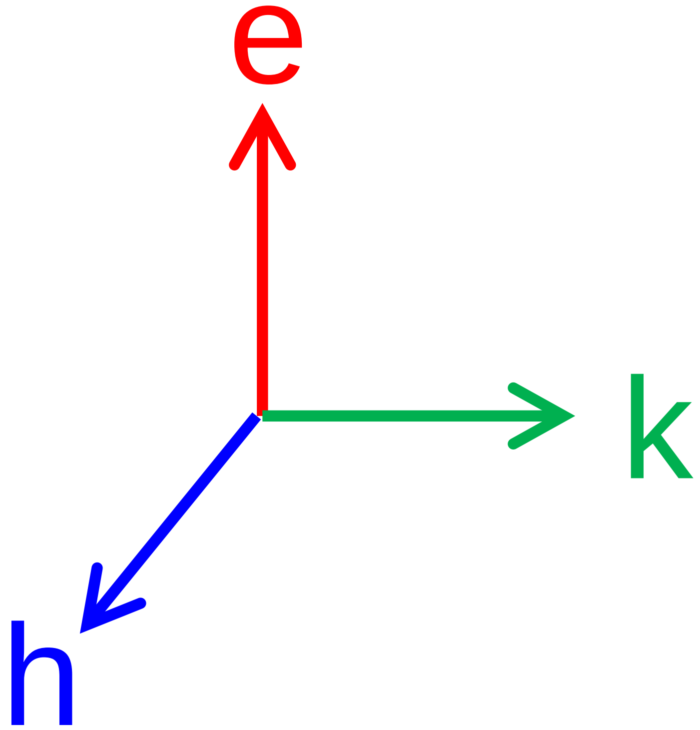
If both T and I broken

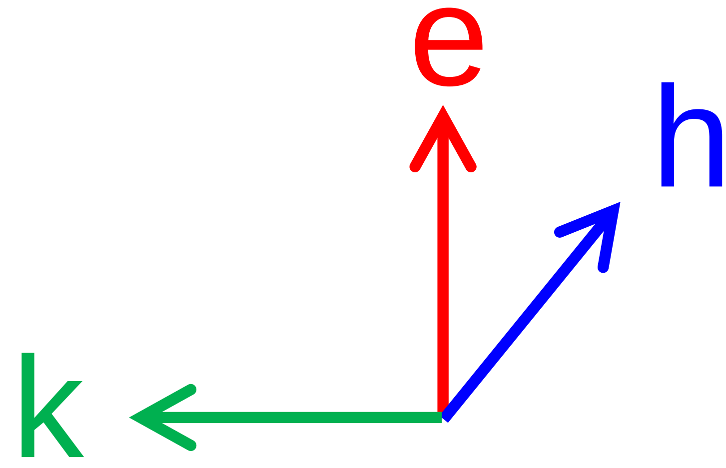
$$\delta n = \gamma \boldsymbol{k} \cdot [\boldsymbol{E} \times \boldsymbol{H}]$$

*N.N. Baranova et al Sov. Phys. Usp. 20, 870 (1977)*

*G. L. J. A. Rikken et al Phys. Rev. Lett. 89, 133005 (2002)*

# Directional dichroism




$$\left| f_e + f_m \right|^2$$

$$\left| f_e - f_m \right|^2$$

# Generators of Pbnm

*Pbnm* (# 62)

| | | |
|---|---|---|
| $E$ | 1 | $x, y, z$ |
| $\tilde{m}_x$ | 2 | $\frac{1}{2}-x, \frac{1}{2}+y, z$ |
| $\tilde{m}_y$ | 3 | $\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z$ |
| $m_z$ | 4 | $x, y, \frac{1}{2}-z$ |
| $I$ | 5 | $\bar{x}, \bar{y}, \bar{z}$ |
| $\tilde{2}_x$ | 6 | $\frac{1}{2}+x, \frac{1}{2}-y, \bar{z}$ |
| $\tilde{2}_y$ | 7 | $\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}-z$ |
| $\tilde{2}_z$ | 8 | $\bar{x}, \bar{y}, \frac{1}{2}+z$ |

$$I = \tilde{m}_x \tilde{m}_y m_z - (0,1,1)$$

$$\tilde{2}_x = \tilde{m}_x I$$

$$\tilde{2}_y = \tilde{m}_y I$$

$$\tilde{2}_z = m_z I$$

http://img.chem.ucl.ac.uk/sgp

http://www.cryst.ehu.es/

# Symmetries of the *ab* spiral

| | $A_x$ | $A_y$ | $A_z$ | $\cos Qy$ | $\sin Qy$ |
|---|---|---|---|---|---|
| $\tilde{m}_y$ | + | − | + | + | − |
| $m_z$ | + | + | − | + | + |

$$\langle \boldsymbol{S} \rangle = \left( A_x \cos Qy, A_y \sin Qy, 0 \right)$$

**Time-reversal symmetry:** $\boldsymbol{S} \to -\boldsymbol{S}$

$$T + S_y\left(\frac{\pi}{Q}\right)$$

# Symmetry of (electro)magnons

| | $e_x$ | $e_y$ | $e_z$ | $h_x$ | $h_y$ | $h_z$ |
|---|---|---|---|---|---|---|
| $\tilde{m}_y$ | + | − | + | − | + | − |
| $m_z$ | + | + | − | − | − | + |

$(+,-)$ $\boldsymbol{e_z}$ & $\boldsymbol{h_y}$ $\delta\mathbf{S} = (0,0,\delta A_z \cos Qy)$

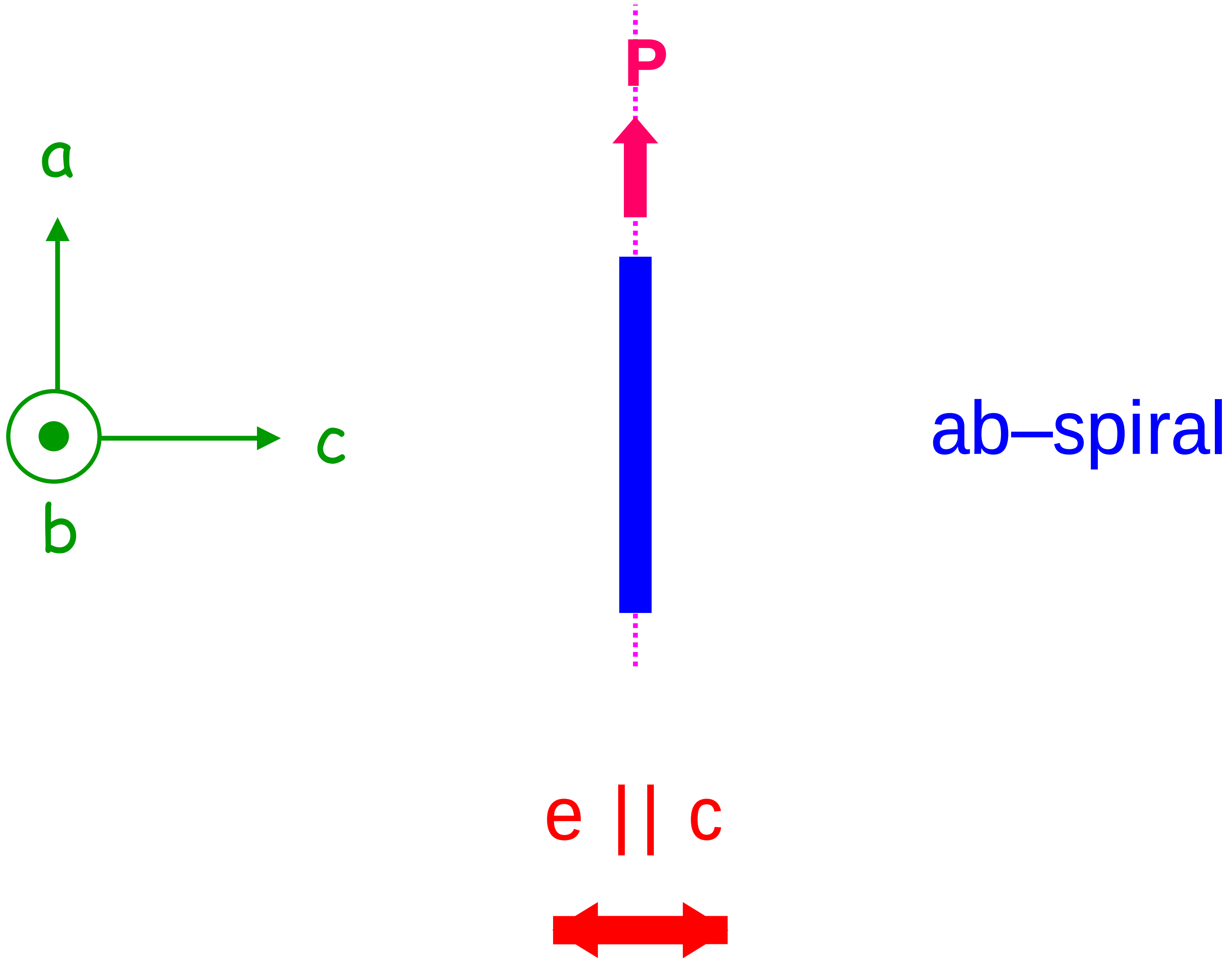
$(-,-)$ $\boldsymbol{h_x}$ $\delta\mathbf{S} = (0,0,\delta A_z \sin Qy)$

$(-,+)$ $\boldsymbol{h_z}$ **phason**

$$\delta\mathbf{S} = \left(-\delta A_x \sin Qy, \delta A_y \cos Qy, 0\right)$$

# Photo-excitation of magnons

***Katsura, Balatsky & Nagaosa (2006)***



# Symmetry of (electro)magnons

| | $e_x$ | $e_y$ | $e_z$ | $h_x$ | $h_y$ | $h_z$ |
|---|---|---|---|---|---|---|
| $\tilde{m}_y$ | + | − | + | − | + | − |
| $m_z$ | + | + | − | − | − | + |

$(+,-)$ $\boldsymbol{e_z}$ & $\boldsymbol{h_y}$ $\delta\mathbf{S} = (0,0,\delta A_z \cos Qy\,)$

$(-,-)$ $\boldsymbol{h_x}$ $\delta\mathbf{S} = (0,0,\delta A_z \sin Qy\,)$

$(-,+)$ $\boldsymbol{h_z}$ **phason**

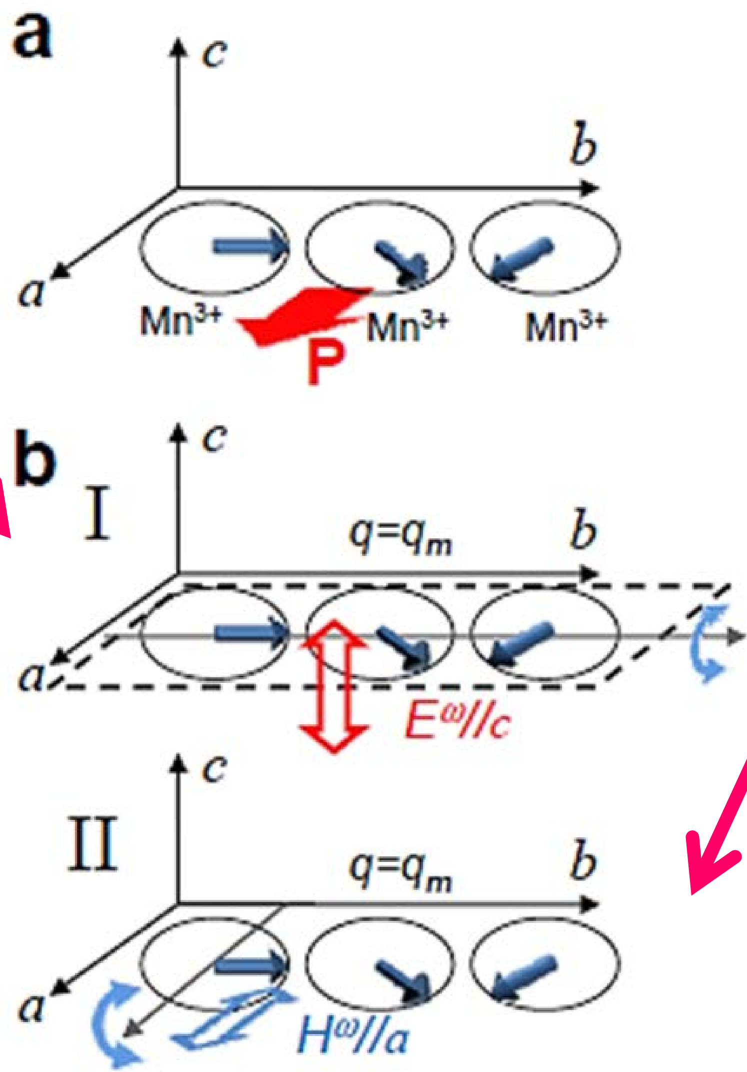
$$\delta\mathbf{S} = \left(-\delta A_x \sin Qy, \delta A_y \cos Qy, 0\right)$$

# Collective modes in spiral state

$\boxed{e^{iQy} + e^{-iQy}}$

e-magnon

e || c




$\boxed{e^{iQy} - e^{-iQy}}$

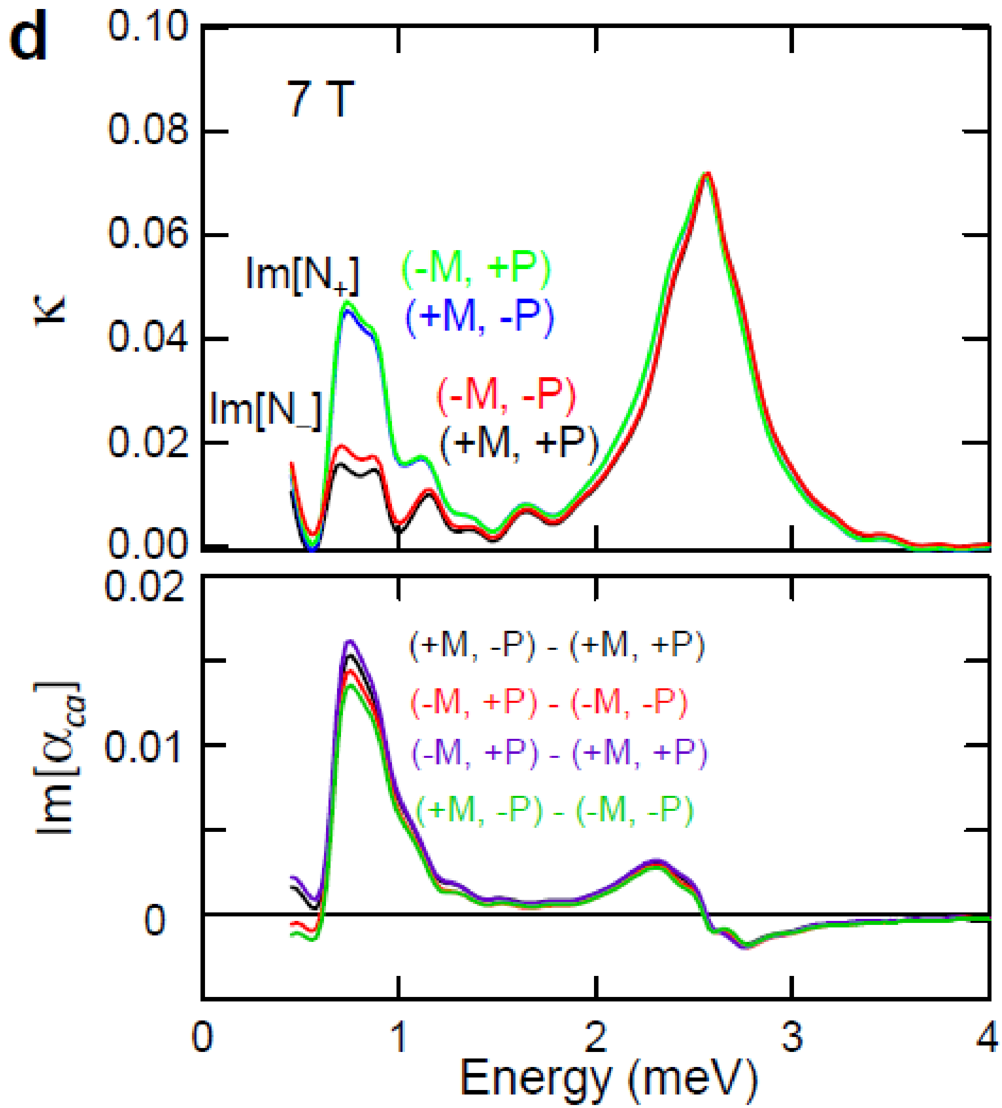
AFMR

h || b

h || a

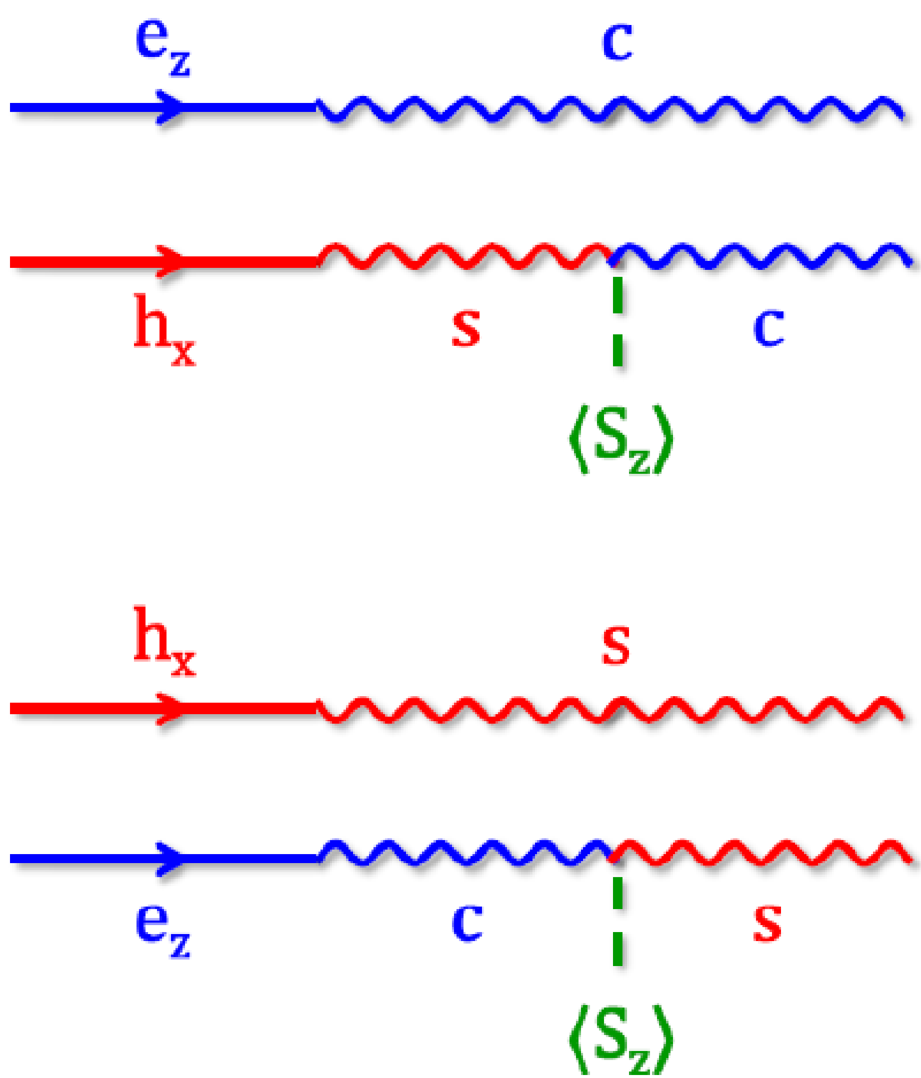
*Y. Takahashi et al, Nature Physics (2011)*

# Directional dichroism in (EuY)$MnO_3$




*Y. Takahashi et al, Nature Physics (2011)*

# Interference between e-magnon and AFM resonance




*A. Cano & MM*

# Directional dichroism and dynamical ME effect

**Refraction index**

$$N_{\pm} = n_{\pm} + i\kappa_{\pm}$$

**Directional dichroism**

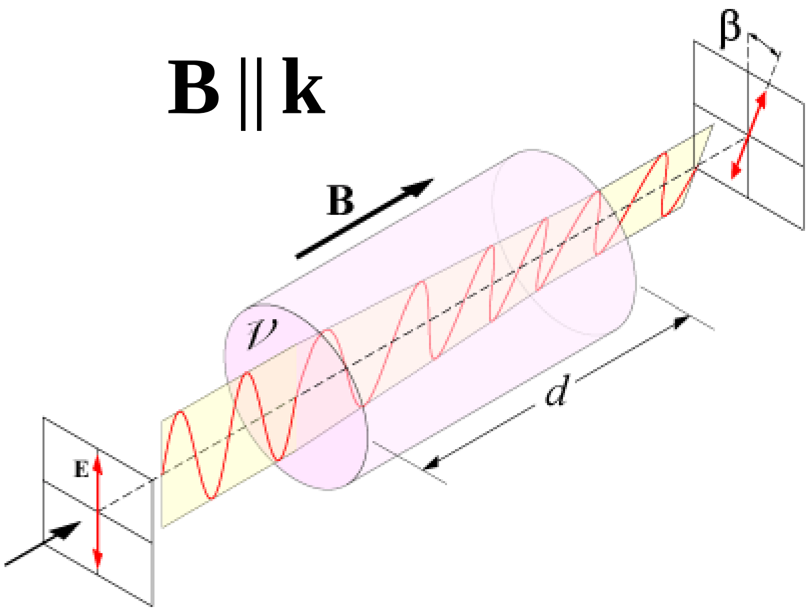
$$\kappa_{+} - \kappa_{-} = \mathrm{Im}\left(\chi_{zx}^{em} + \chi_{xz}^{me}\right)$$

$$\delta P_z = \chi_{zx}^{em} h_x \qquad \delta M_x = \chi_{xz}^{me} e_z$$

# Faraday Effect

$\mathbf{B} \parallel \mathbf{k}$

$\mathbf{e} \perp \mathbf{k}$




$$\left(\mathbf{k}\cdot\nabla\right)\mathbf{e} = C\left[\mathbf{B}\times\mathbf{e}\right]$$

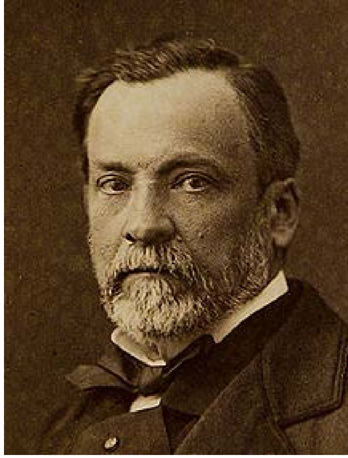
$\mathbf{B},\mathbf{k} \parallel \mathbf{z}$

$$k_z \frac{de_x}{dz} = +CB_z e_y$$

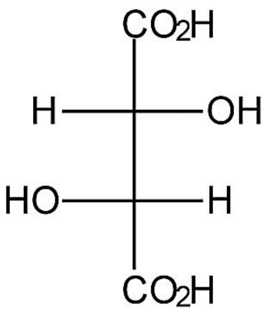
$$k_z \frac{de_y}{dz} = -CB_z e_x$$

Jean Baptiste Biot
(1774 – 1862)

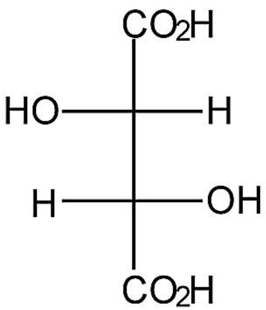
# Natural Optical Activity



Louis Pasteur
(1822 – 1895)




L(+) tartaric acid



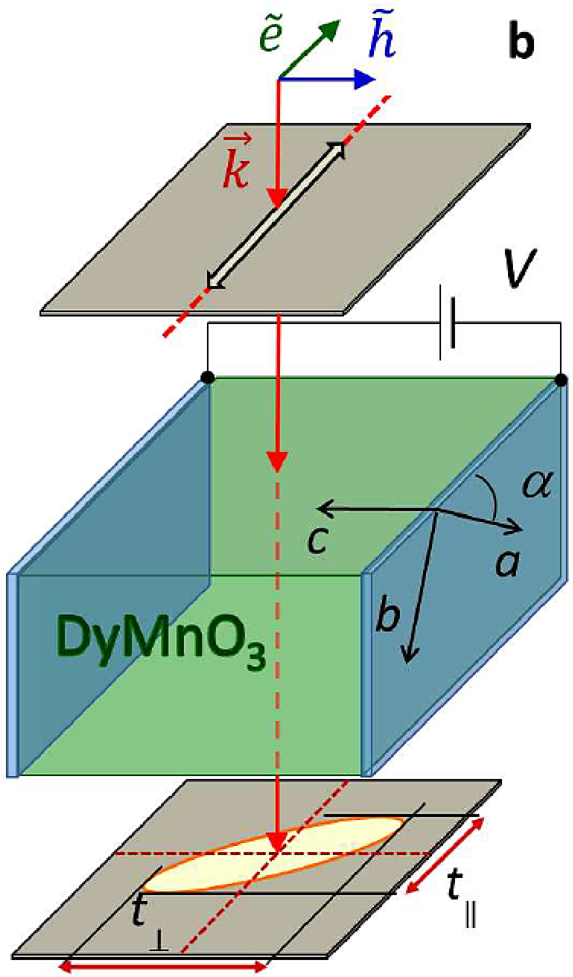

D(-) tartaric acid

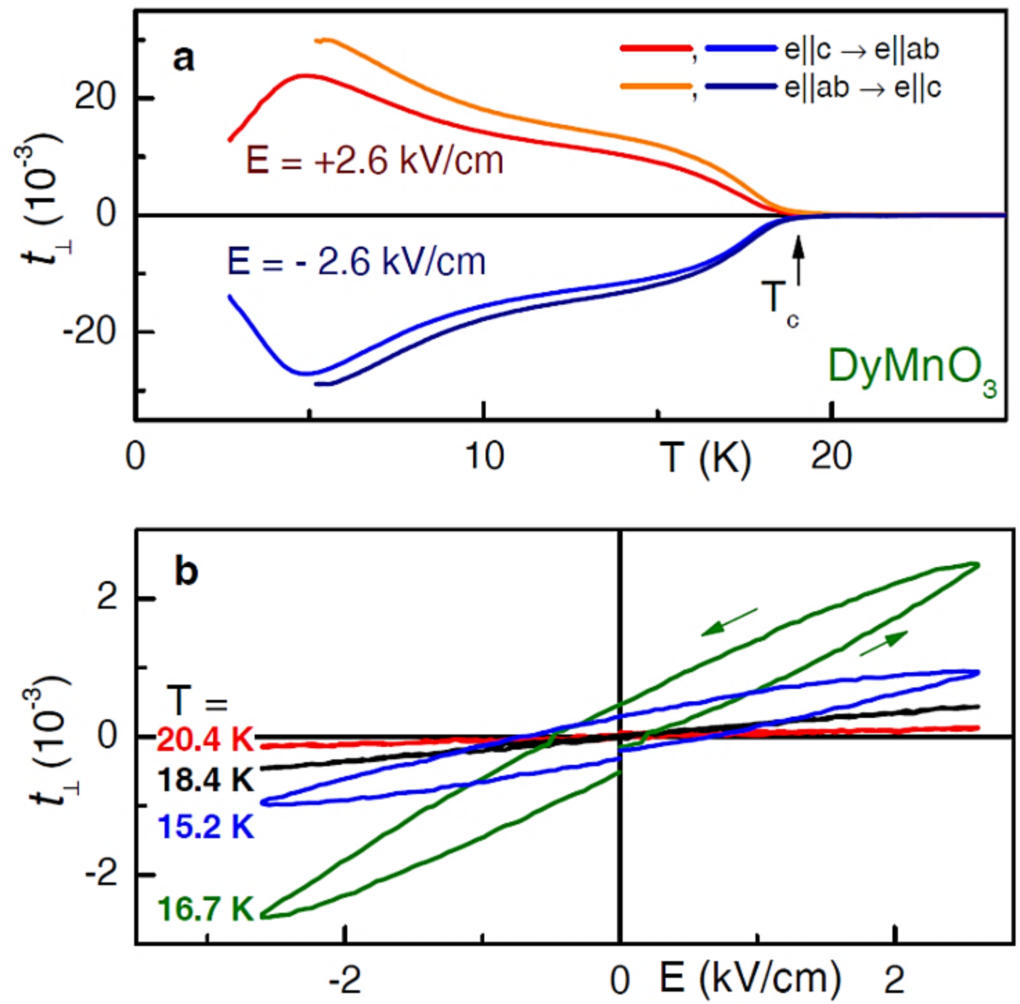
$$\frac{de_x}{dz} = +Ce_y$$

$$\frac{de_y}{dz} = -Ce_x$$

Under inversion $\mathbf{E} \to -\mathbf{E},\ z \to -z$ equations change sign $C = -C = 0$

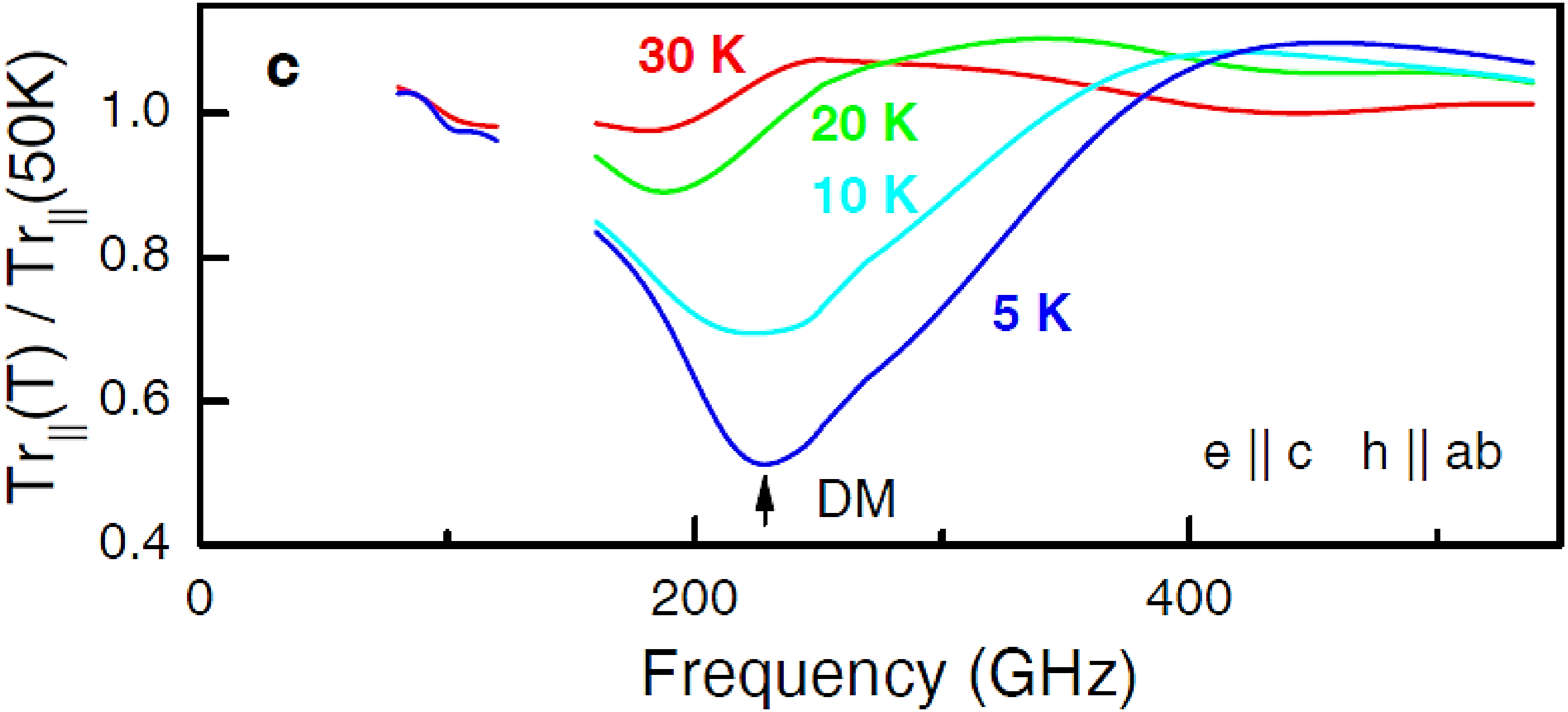
# Electric field control of light polarization in $DyMnO_3$




*Shuvaev et al PRL 111, 227201 (2017)*

# Electromagnon




*Shuvaev et al PRL 111, 227201 (2017)*

# Symmetries of the *bc* spiral

| | $A_x$ | $A_y$ | $A_z$ | $\cos Qy$ | $\sin Qy$ |
|---|---|---|---|---|---|
| $\tilde{m}_y$ | + | − | + | + | − |
| $\tilde{2}_z + S_y$ | − | − | + | + | − |

$$\langle \boldsymbol{S} \rangle = \left(0, A_y \sin Qy, A_z \cos Qy\right)$$

**Time-reversal symmetry:** $\boldsymbol{S} \to -\boldsymbol{S}$

$$T + S_y\left(\frac{\pi}{Q}\right)$$

# Symmetry of (electro)magnons

| | $e_x$ | $e_y$ | $e_z$ | $h_x$ | $h_y$ | $h_z$ |
|---|---|---|---|---|---|---|
| $\tilde{m}_y$ | + | − | + | − | + | − |
| $\tilde{2}_z$+$S_y$ | − | − | + | − | − | + |

$(+,-)$ $\boldsymbol{e_x}$ & $\boldsymbol{h_y}$ $\delta\mathbf{S} = (\delta A_x \cos Qy, 0,0\ )$

$(-,+)$ $\boldsymbol{h_z}$ $\delta\mathbf{S} = (\delta A_x \sin Qy, 0,0\ )$

$(-,-)$ $\boldsymbol{h_x}$ **phason**

$$\delta\mathbf{S} = \left(0, \delta A_y \cos Qy, -\delta A_z \sin Qy\right)$$

# Susceptibilities

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} \qquad \mu_{ij} = \begin{pmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{pmatrix}$$

$$\alpha_{ik}^{em} = \begin{pmatrix} 0 & \alpha_{xy}^{em} & 0 \\ \alpha_{yx}^{em} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \alpha_{ik}^{me} = \begin{pmatrix} 0 & \alpha_{xy}^{me} & 0 \\ \alpha_{yx}^{me} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

**Time reversal invariance**

$$\alpha_{ik}^{me} = -\alpha_{ki}^{em}$$

# Maxwell equations

$$\boldsymbol{b} = +\boldsymbol{n} \times \boldsymbol{e}$$

$$\boldsymbol{d} = -\boldsymbol{n} \times \boldsymbol{h}$$

$$\boldsymbol{k} = \frac{\omega}{c}\boldsymbol{n}$$

$$d_i = \varepsilon_{ik} e_k + \alpha_{ik}^{em} h_k$$

$$b_i = \alpha_{ik}^{me} e_k + \mu_{ik} h_k$$

# Fresnel equation

$$\left(\frac{n^2}{\varepsilon(\varphi)\mu_{zz}} - 1\right)\left(\frac{n^2}{\mu(\varphi)\varepsilon_{zz}} - 1\right)$$

$$+\frac{\alpha^2}{\varepsilon_{xx}\mu_{yy}}\left(\frac{n^2\cos^2\varphi}{\varepsilon_{yy}\mu_{zz}} - 1\right)\left(\frac{n^2\sin^2\varphi}{\varepsilon_{zz}\mu_{xx}} - 1\right) = 0$$

$$\frac{1}{\varepsilon(\varphi)} = \frac{\sin^2\varphi}{\varepsilon_{xx}} + \frac{\cos^2\varphi}{\varepsilon_{yy}} \qquad \frac{1}{\mu(\varphi)} = \frac{\sin^2\varphi}{\mu_{xx}} + \frac{\cos^2\varphi}{\mu_{yy}}$$

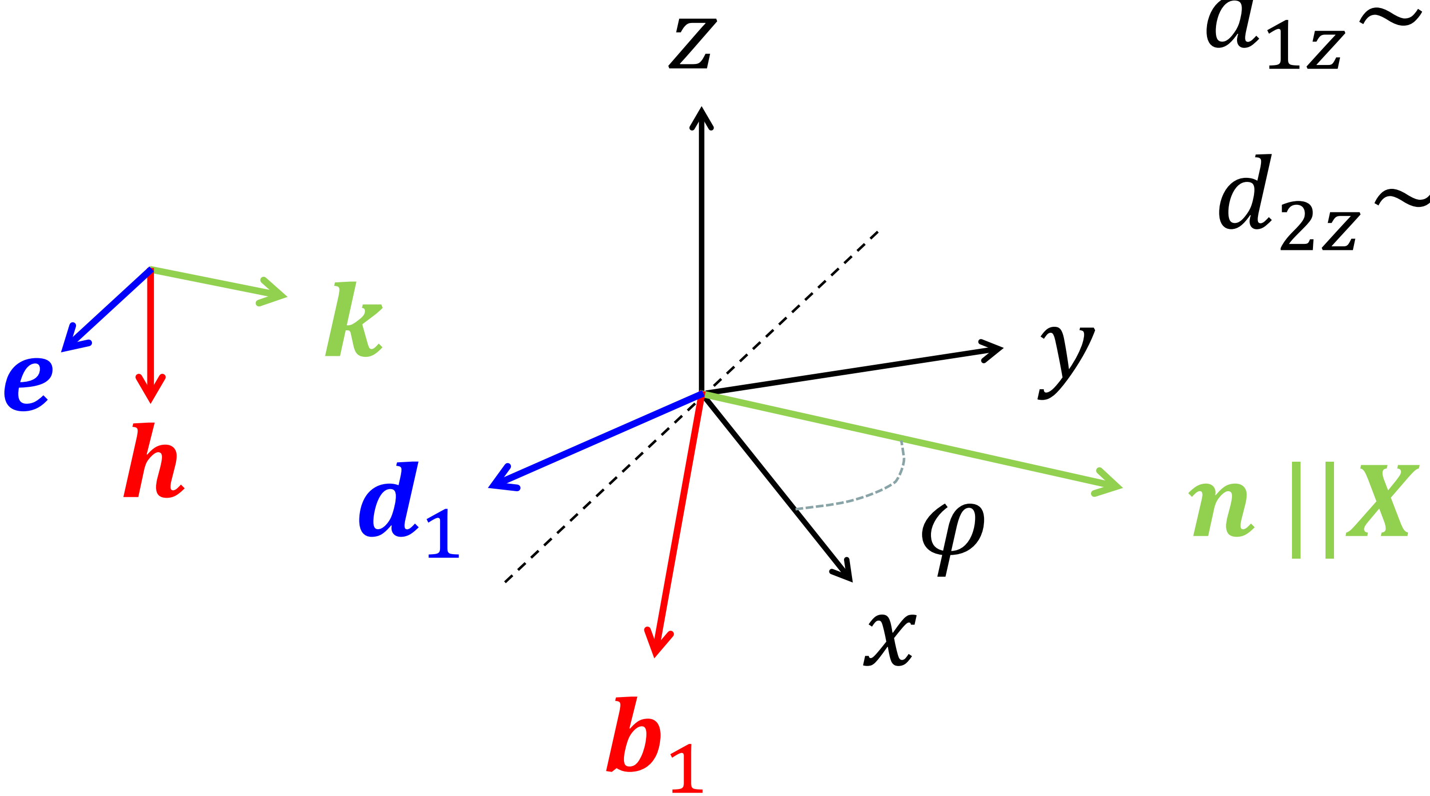
$$\alpha = \alpha_{xy}^{em} = -\alpha_{yx}^{me} \qquad \alpha_{yx}^{em} = -\alpha_{xy}^{me} = 0$$

# $d_z$-oscillations due to magnetoelectric coupling

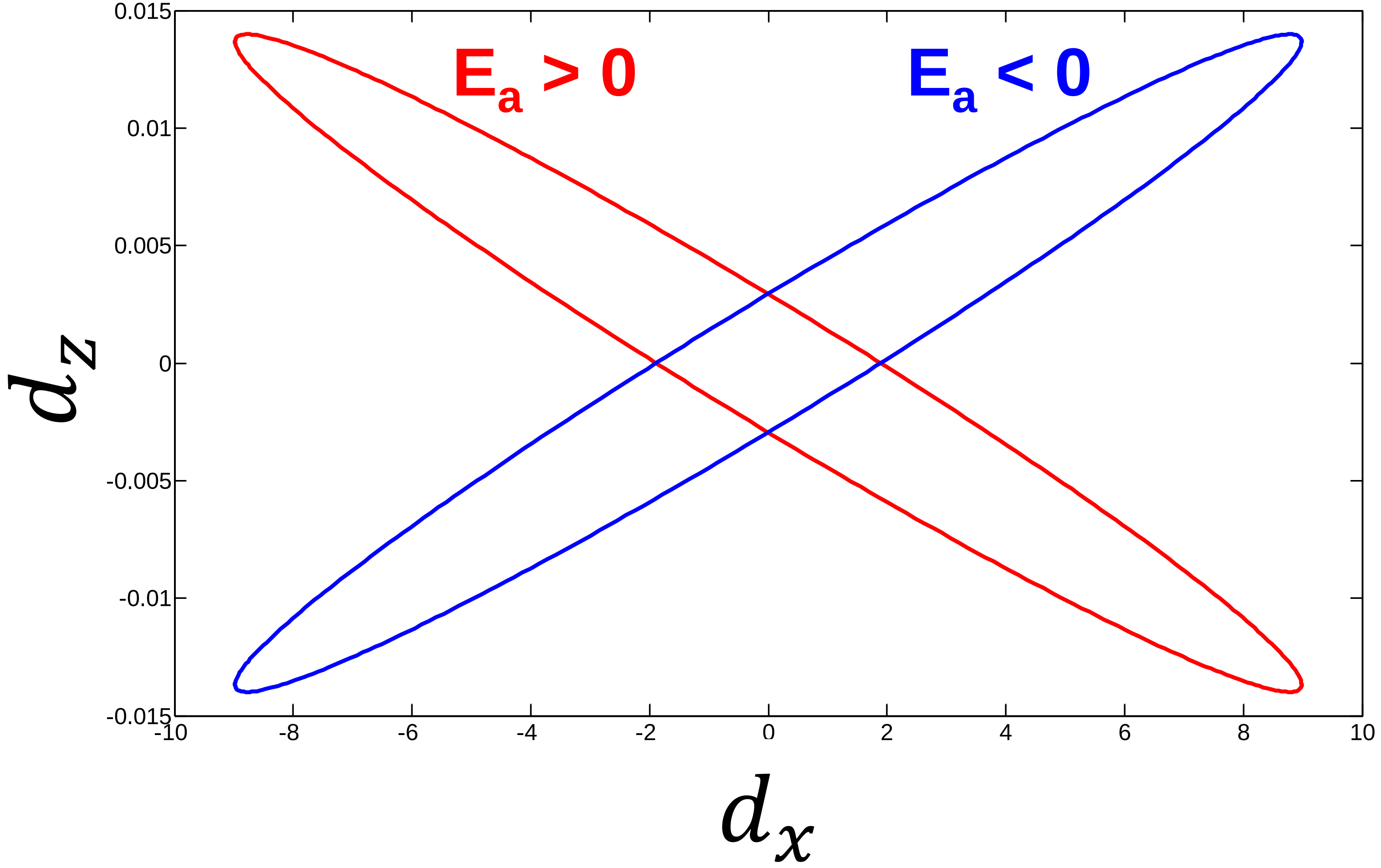
$$d_z = e^{-i\omega t}\left(d_{1z}e^{i\frac{\omega}{c}n_1 X} + d_{2z}e^{i\frac{\omega}{c}n_2 X}\right)$$

$$d_{1z} \sim \alpha$$

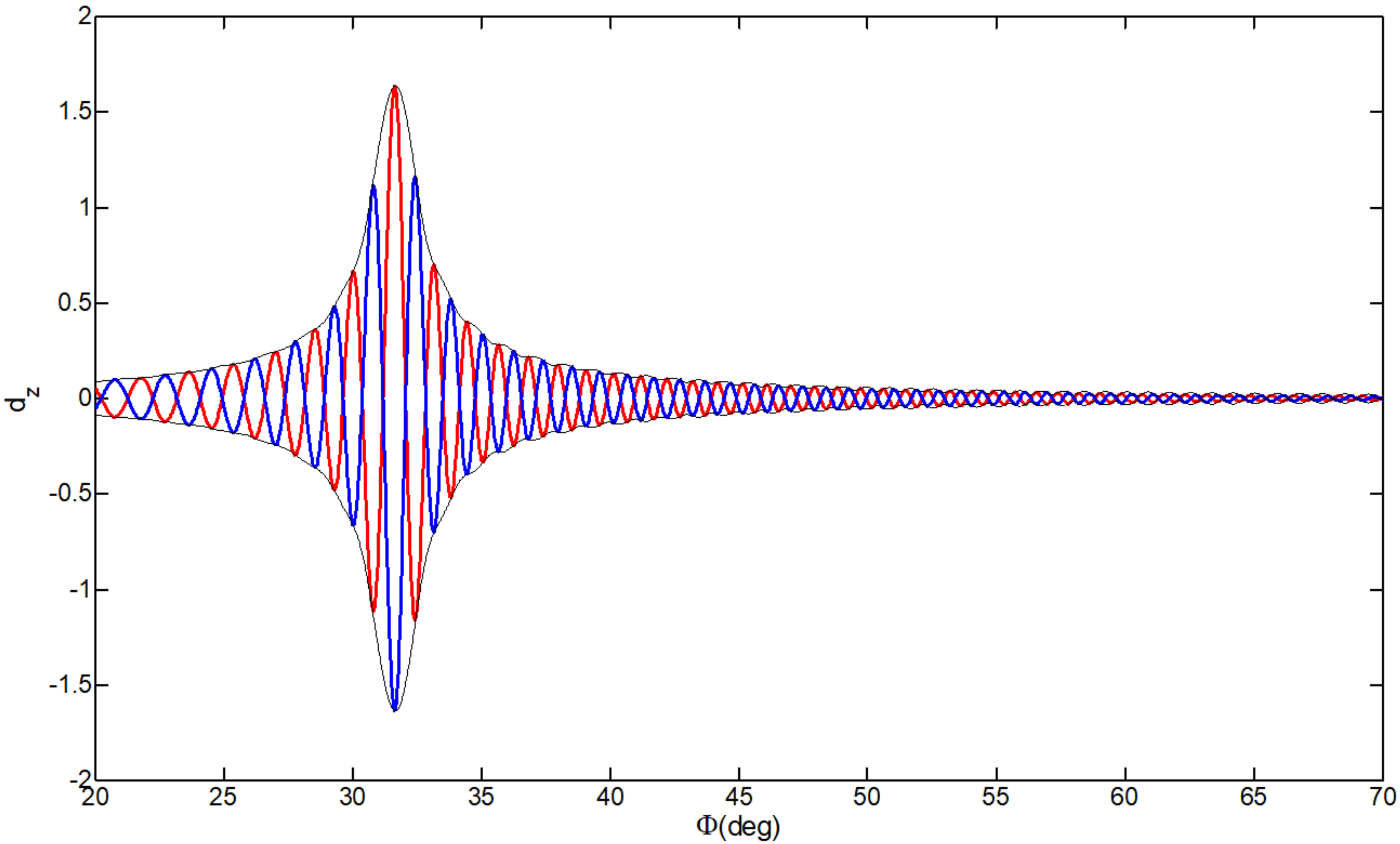
$$d_{2z} \sim \alpha$$



# Elliptical polarization

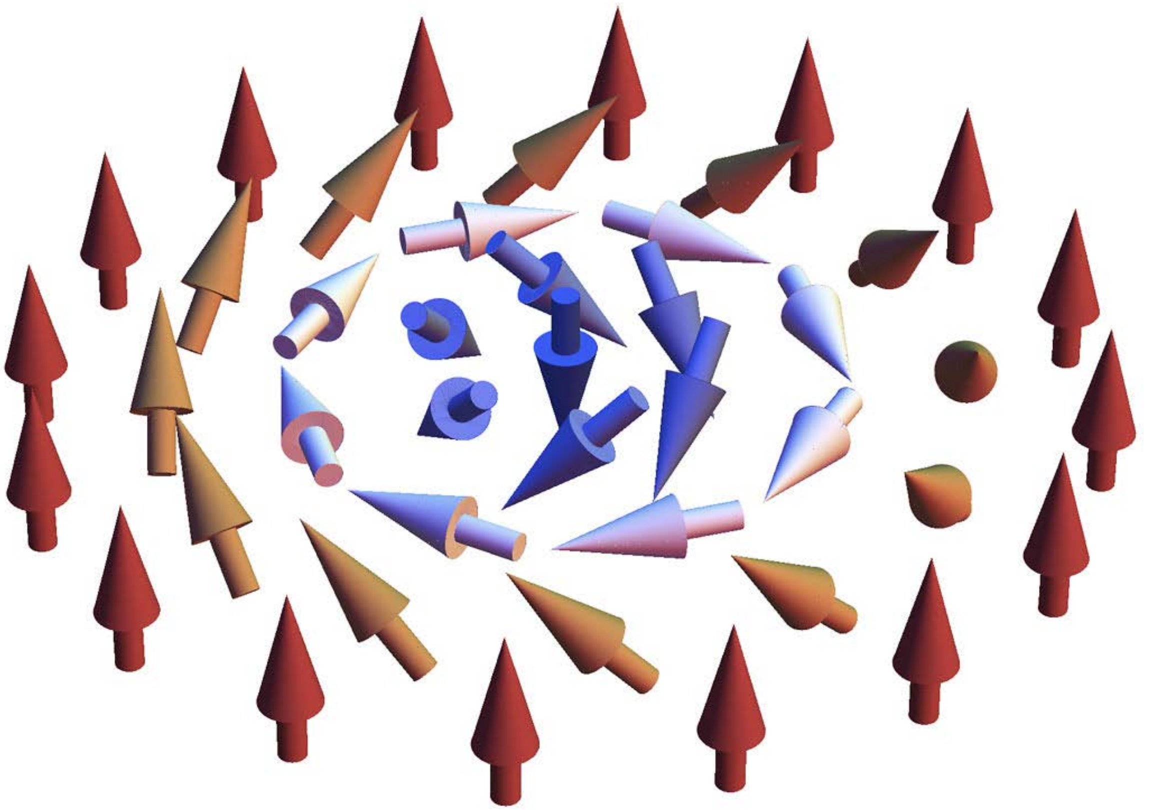


# Enhancement near optical bi-axis



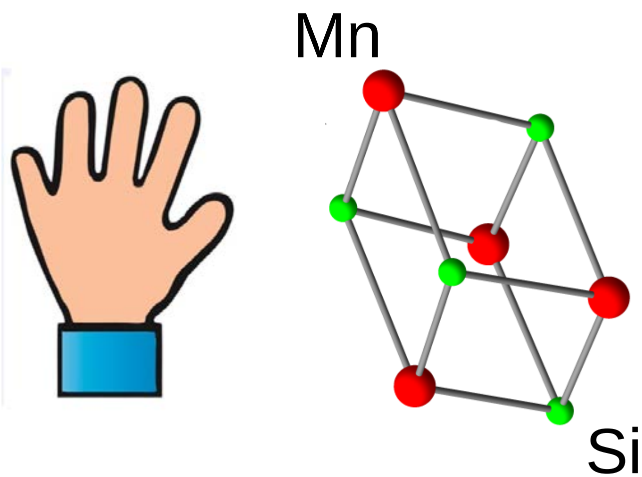
# Magnetic skyrmion

$$n_x^2 + n_y^2 + n_z^2 = 1$$

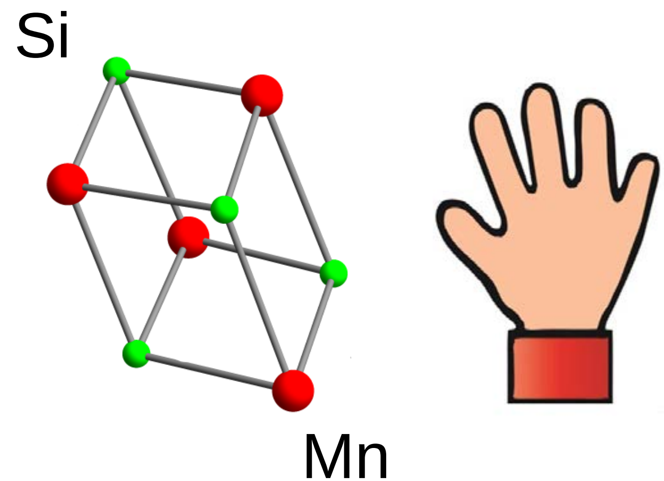


$$S_2 \to S_2$$

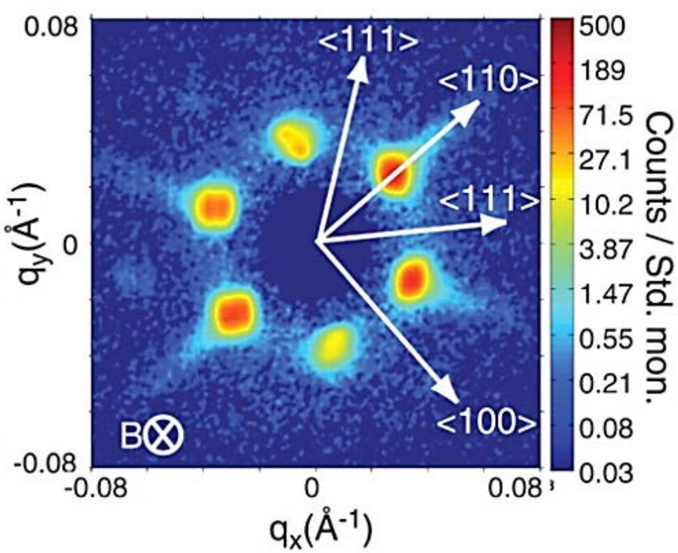
# Skyrmions in chiral magnets




$$\mathbf{x} \to -\mathbf{x}$$

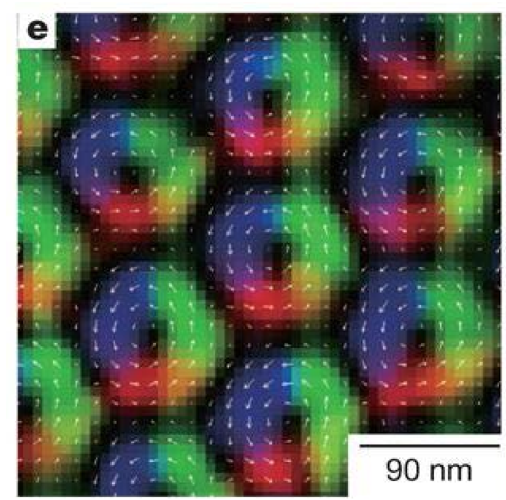



## Neutron scattering




*Mühlbauer et al, Science (2009)*

## Lorentz microscopy




*X. Yu et al. Nature (2010)*

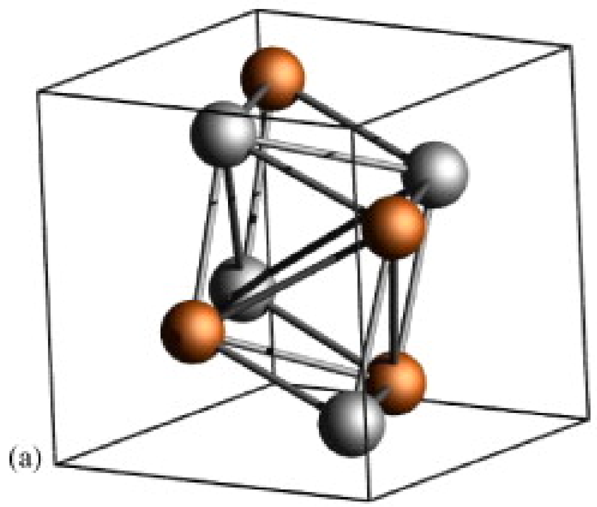
# Non-centrosymmetric magnets

*I. Dzyaloshinskii, Sov. Phys. JETP **19**, 960 (1964)*

*P. Bak & M.H. Jensen, J. Phys. C: Solid State **13**, L881 (1980)*

## Cubic B20 structure $P2_13$

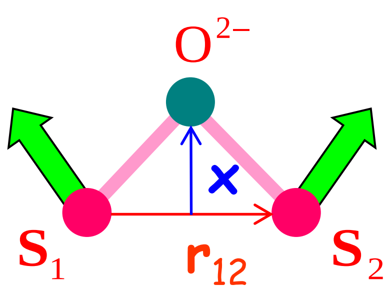
**MnSi, $Fe_{1-x}Co_xSi$, FeGe**




$$f=\frac{J}{2}(\nabla\mathbf{n})^2+D\,\mathbf{n}\cdot[\nabla\times\mathbf{n}]$$

**Exchange energy**

**DM energy**

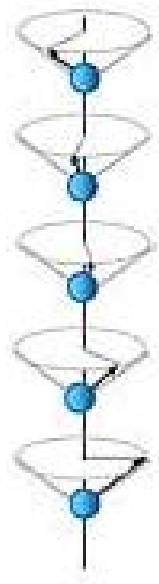


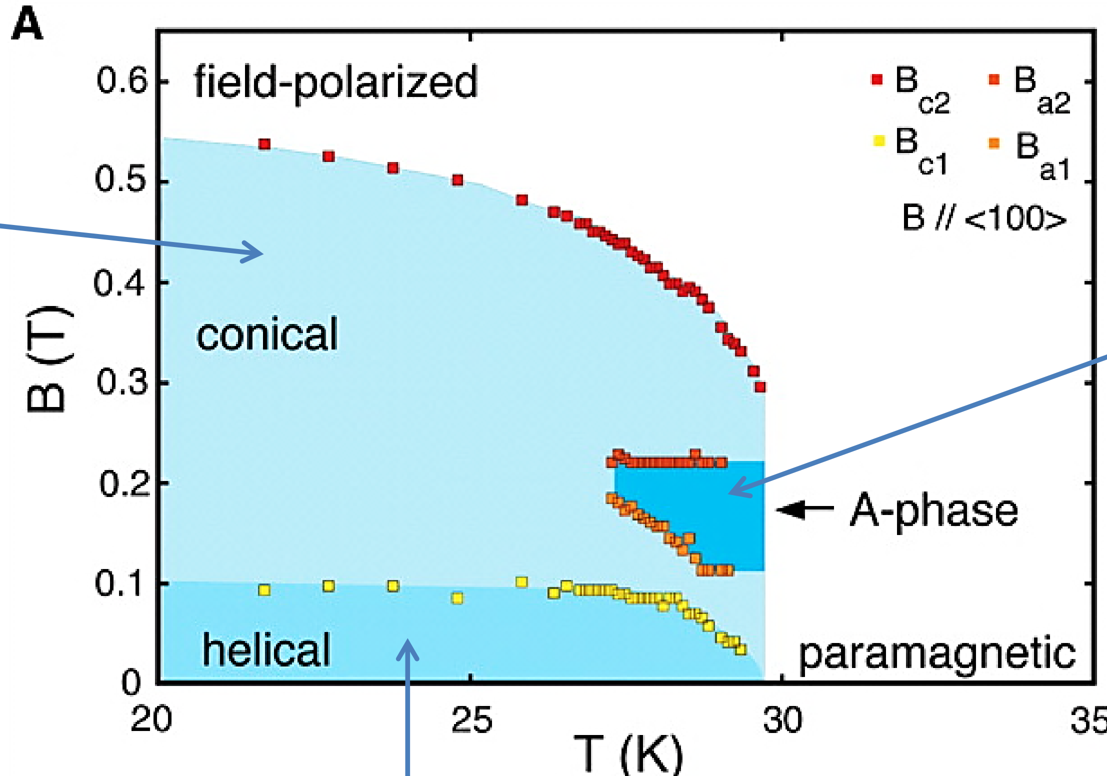

$$E_{DM}=\mathbf{D}_{12}\cdot[\mathbf{S}_1\times\mathbf{S}_2]$$

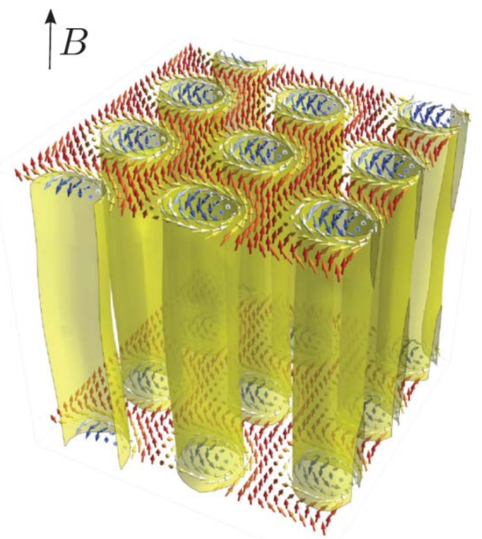
# Phase Diagram of MnSi

*Mühlbauer et al, Science **323**, 915 (2009)*

$\mathbf{q} \parallel \mathbf{H}$





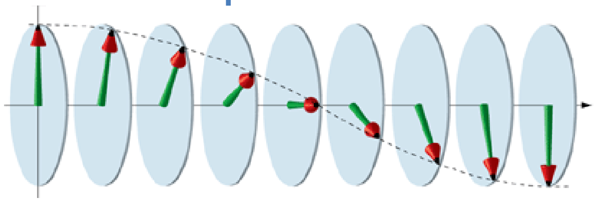

$\mathbf{q}_3$

$\mathbf{q}_1$ $\mathbf{q}_2$

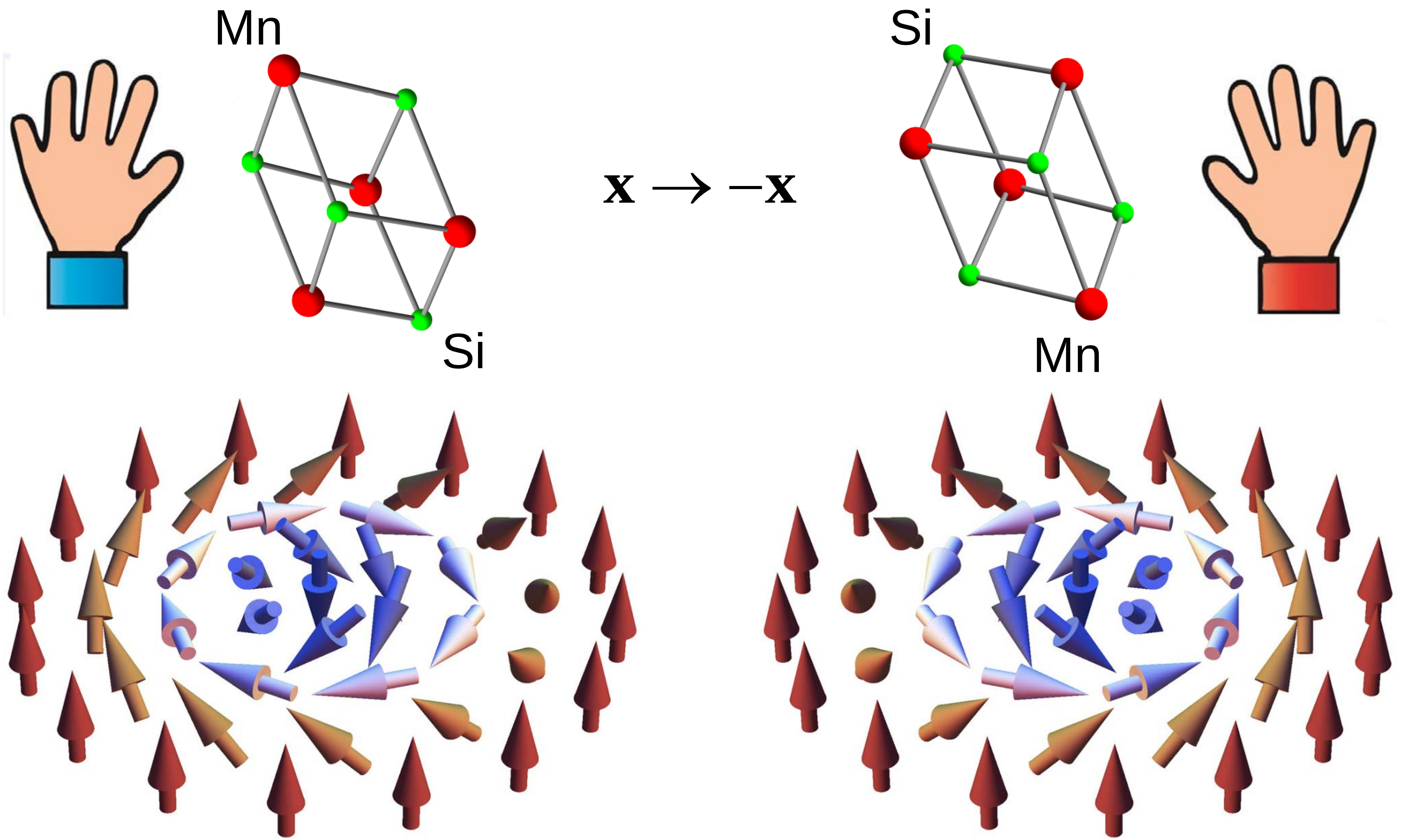
$\mathbf{q} \perp \mathbf{H}$

$\mathbf{q} \parallel [111]$

$\lambda = 18\ \mathrm{nm}$

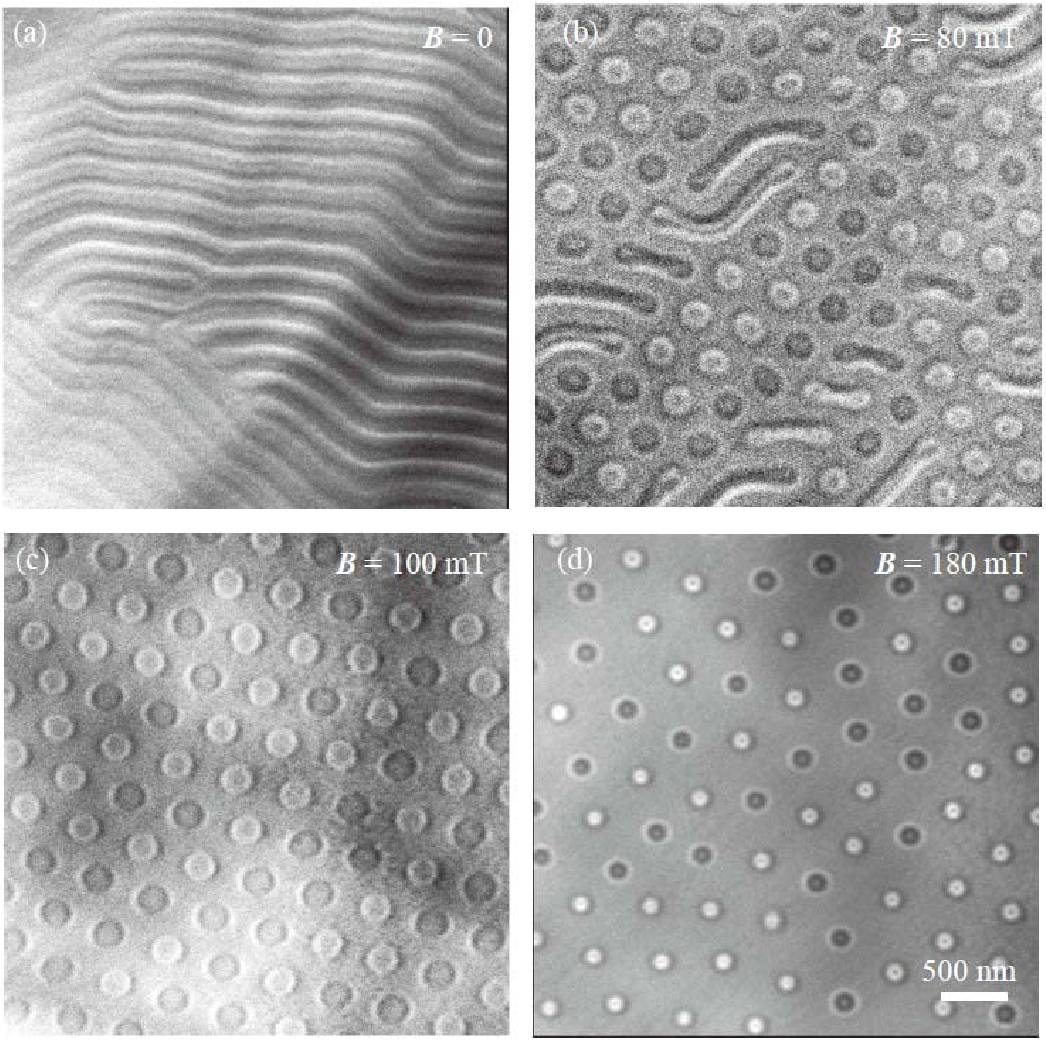


# Skyrmion inversion




Topological charge does not change under inversion

# Stripes & magnetic bubbles



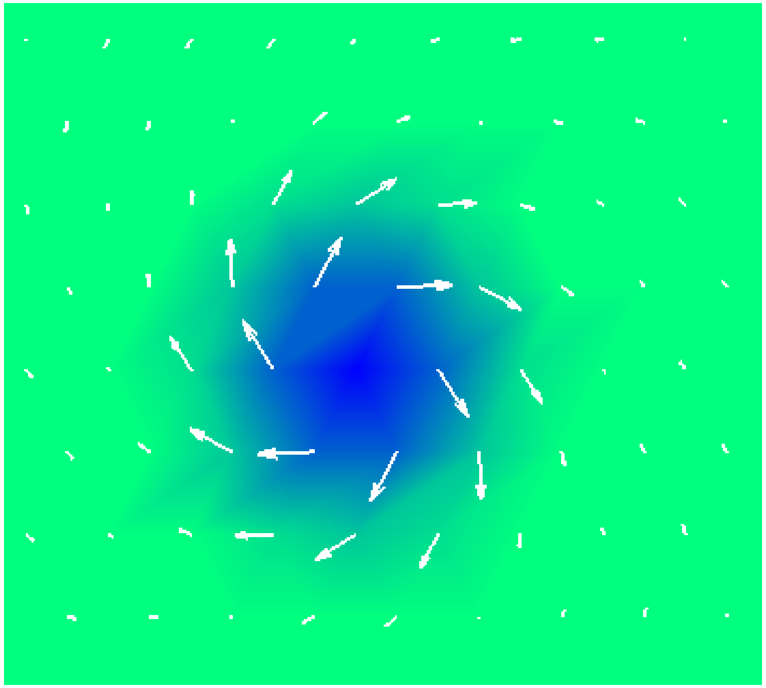

*X. Yu et al. PNAS **109**, 8856 (2012)*

# Skyrmions in frustrated magnets

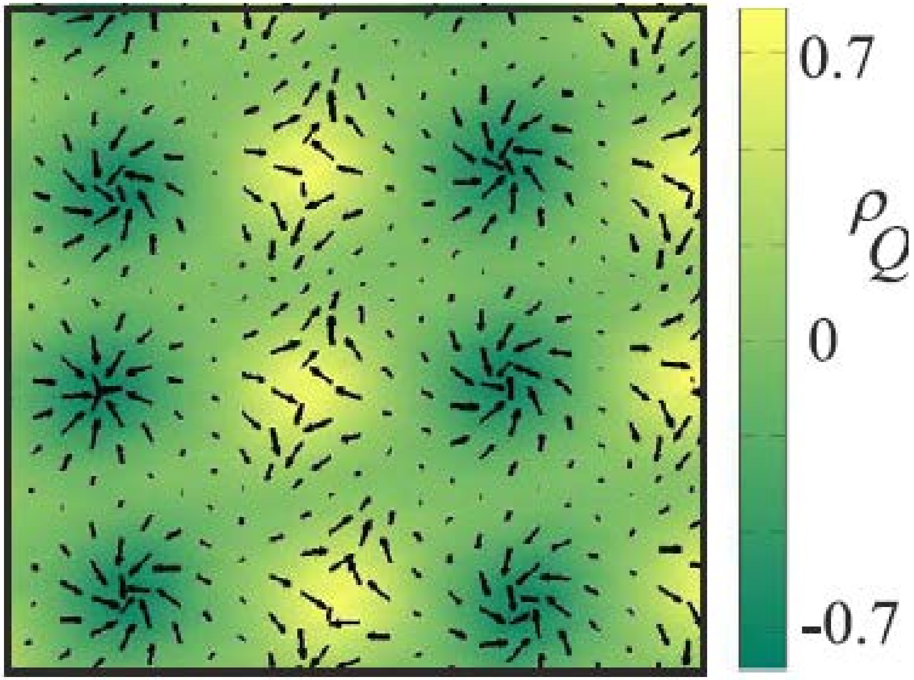
# Arbitrary helicity and vorticity

$$\Theta = \Theta(\rho) \qquad \Phi = \mathrm{v}\,\varphi + \chi$$

$$Q = \mathrm{v}\,\frac{\left(n_z(0) - n_z(\infty)\right)}{2}$$

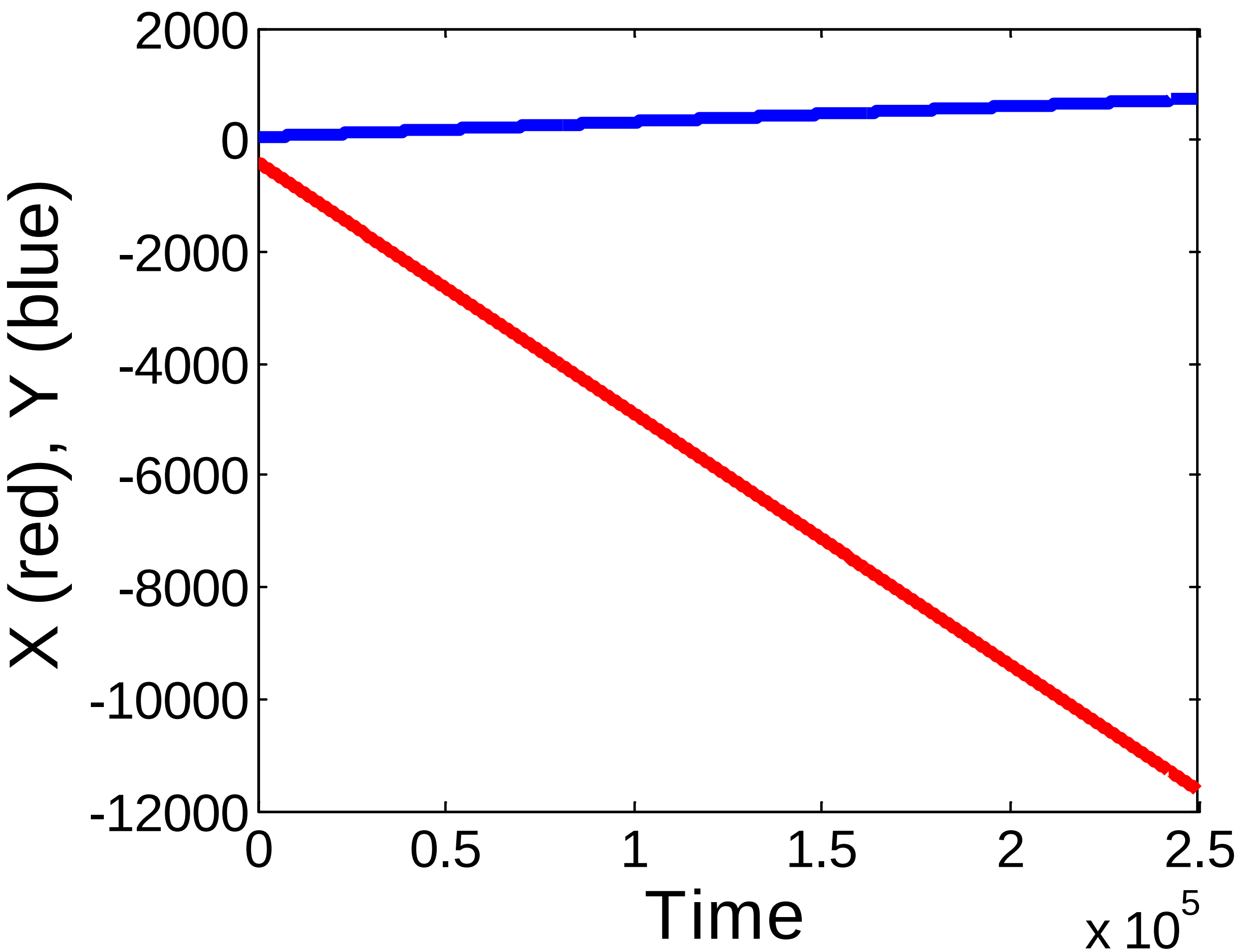


$\boldsymbol{S} \to R_z(\chi)\ \boldsymbol{S}$




$S_x \to -S_x$

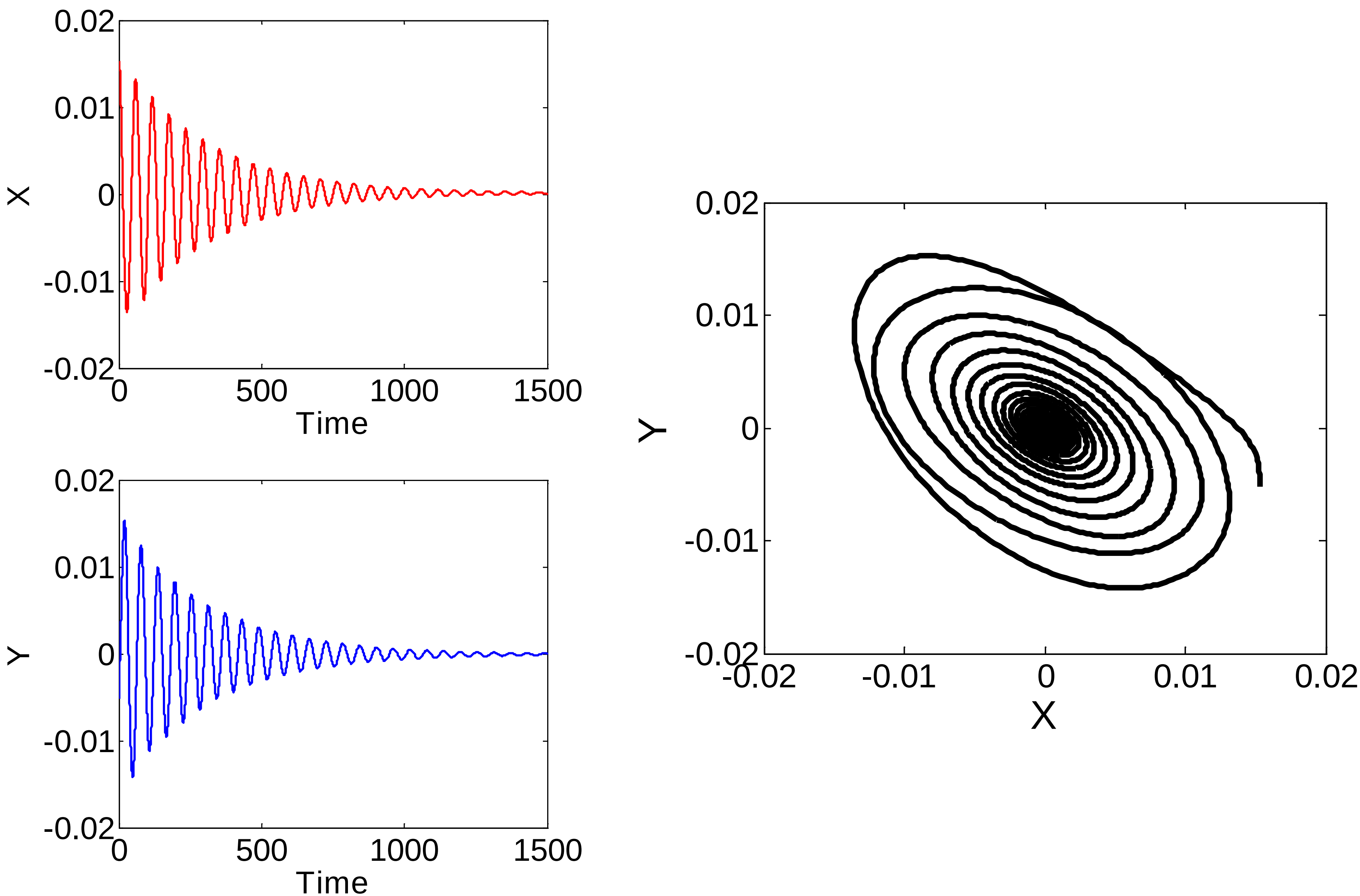
# Current-driven skyrmion: translational motion




$j_x = 0.05$

# Does skyrmion have a mass?

# Coordinate relaxation



# Relaxation of the massive skyrmion

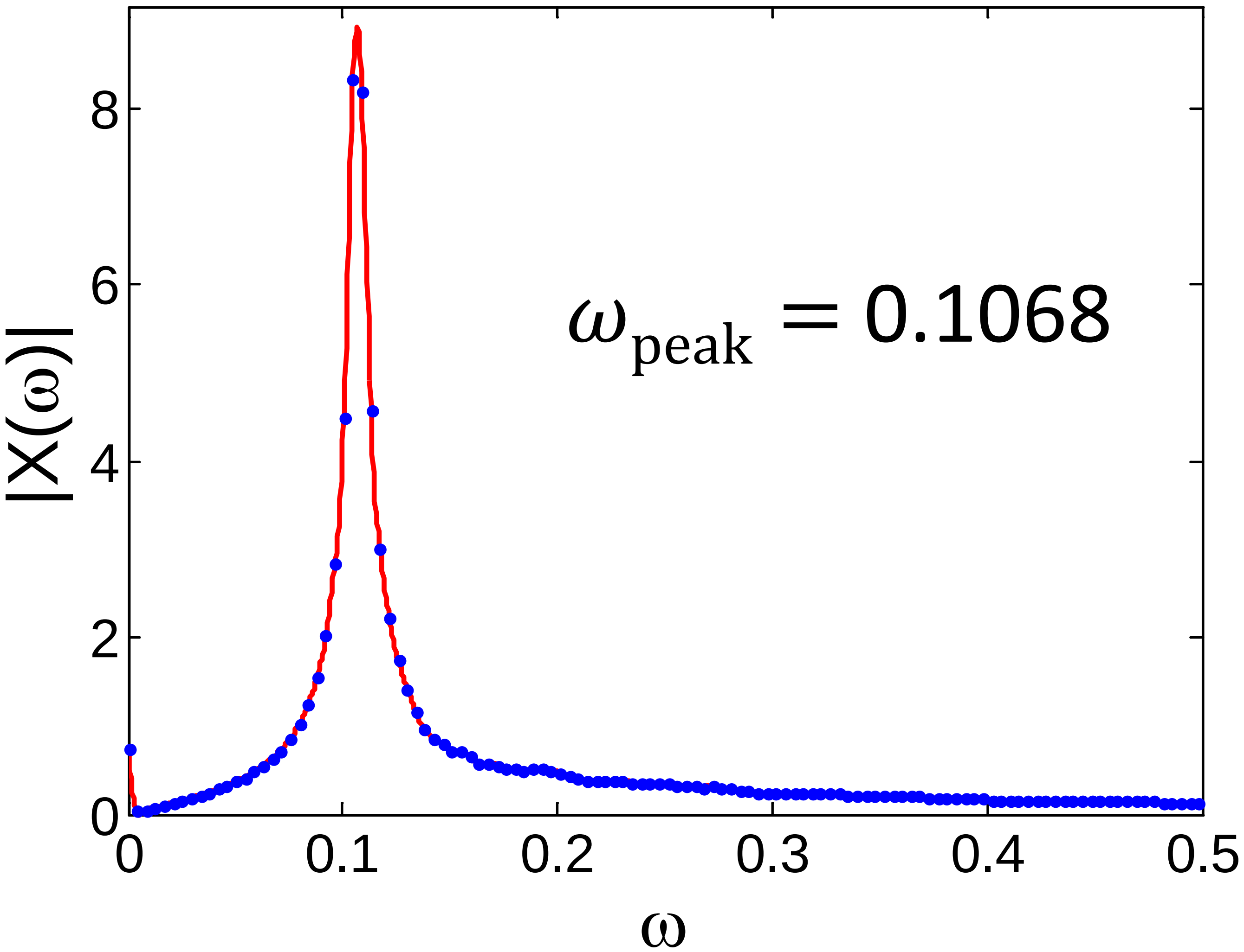
**Center-of-mass coordinates:**

$$\mathbf{m}(\mathbf{x},t)=\mathbf{m}(\mathbf{x}-\mathbf{R}(t)) \qquad \mathbf{R}=(X,Y)$$

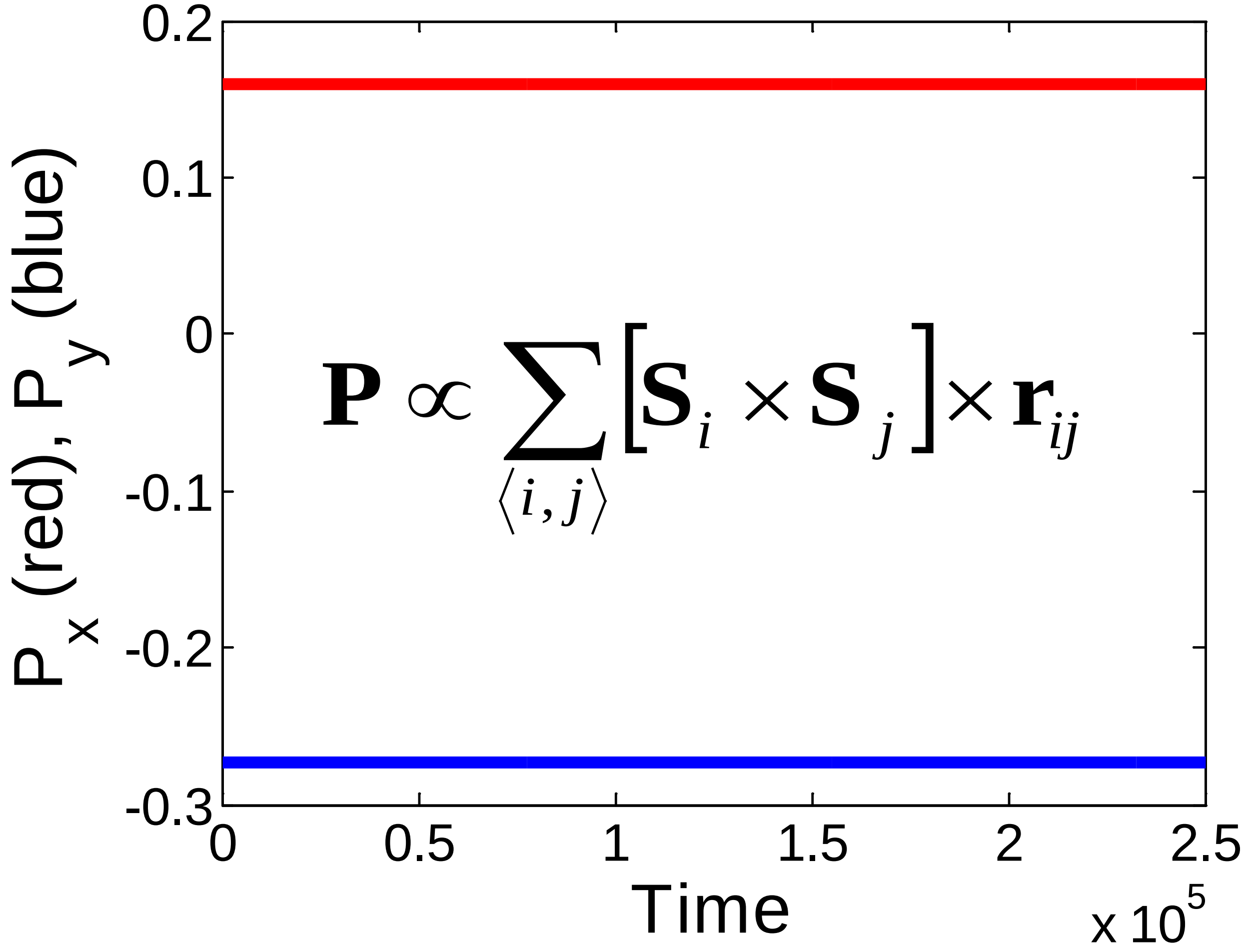
$$\begin{cases} M\ddot{X}+\alpha\Gamma\dot{X}-G\dot{Y}=0 \\ M\ddot{Y}+G\dot{X}+\alpha\Gamma\dot{Y}=0 \end{cases}$$

$$\omega_{\pm}=\frac{\pm|G|-i\alpha\Gamma}{M}$$

# Fourier transform of X(t)



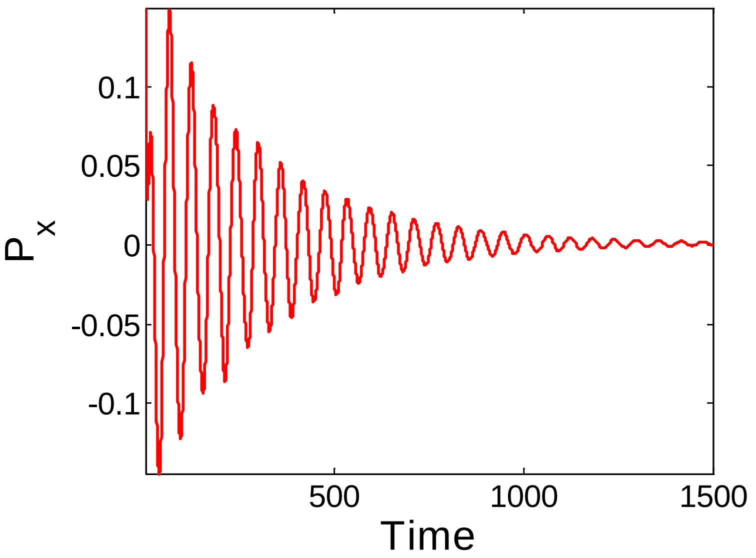
# Current-driven skyrmion: electric polarization

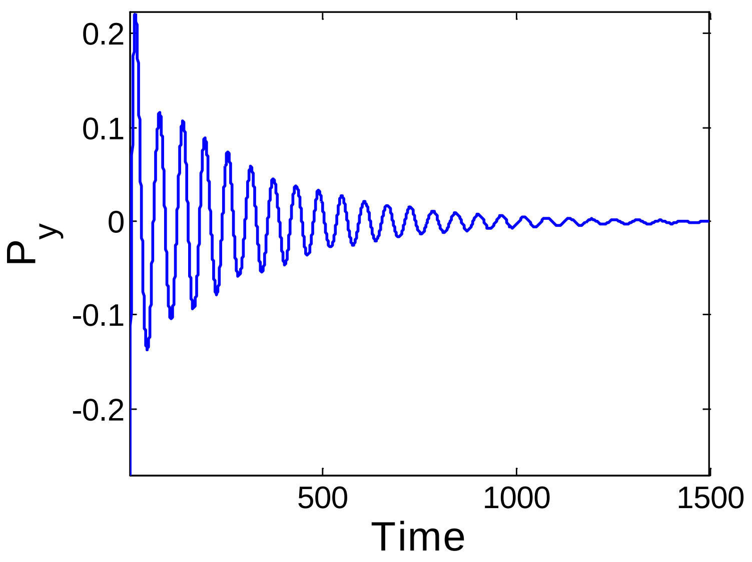


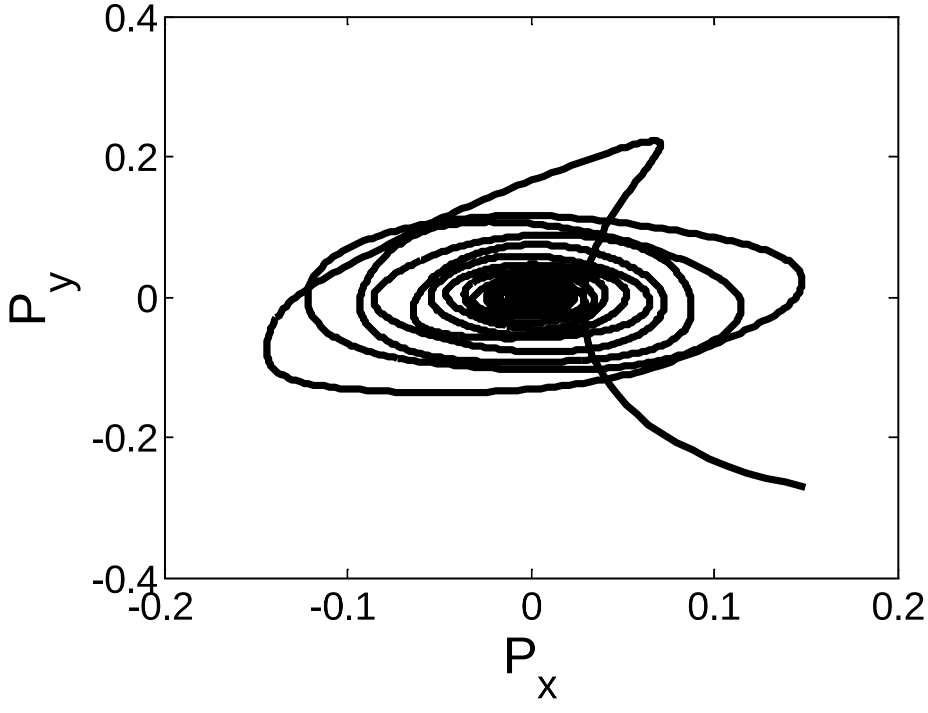

$$\mathbf{P} \propto \sum_{\langle i,j \rangle} \left[ \mathbf{S}_i \times \mathbf{S}_j \right] \times \mathbf{r}_{ij}$$

$j_x = 0.05$

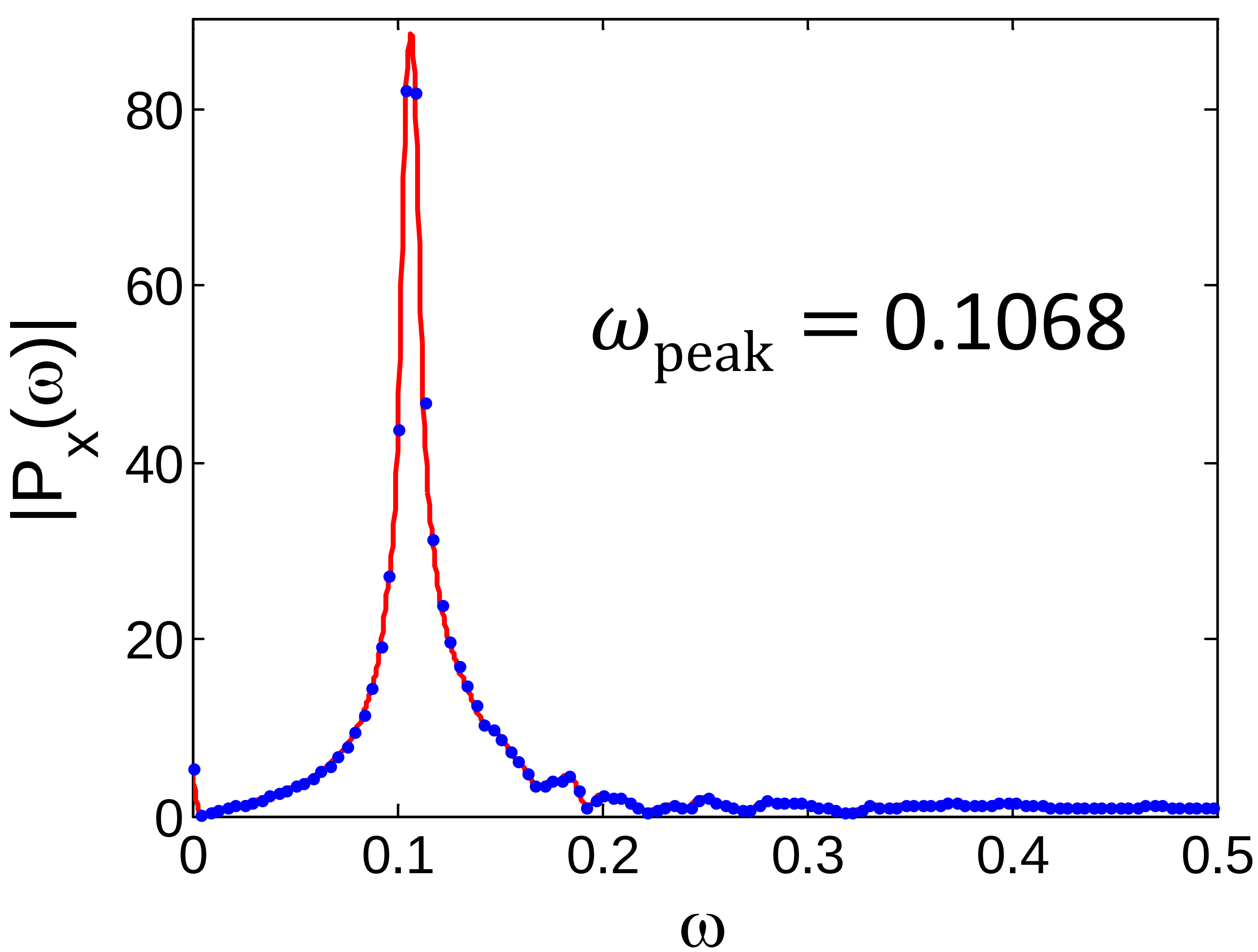
# Electric polarization relaxation



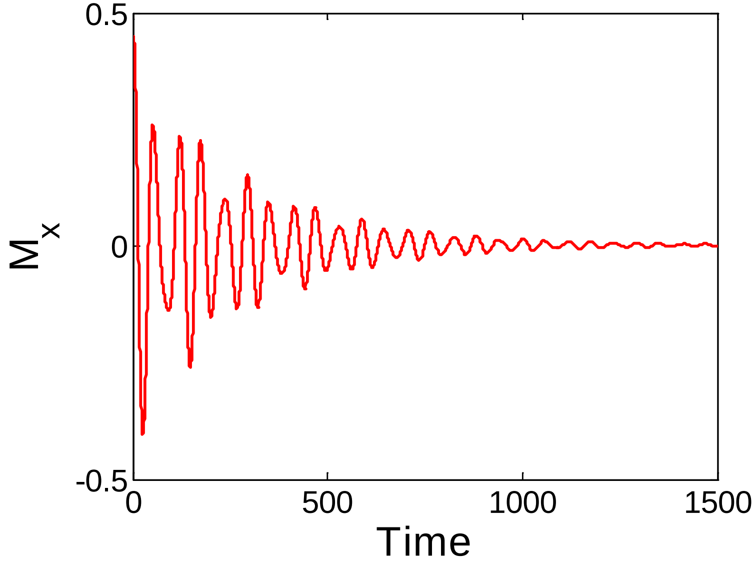



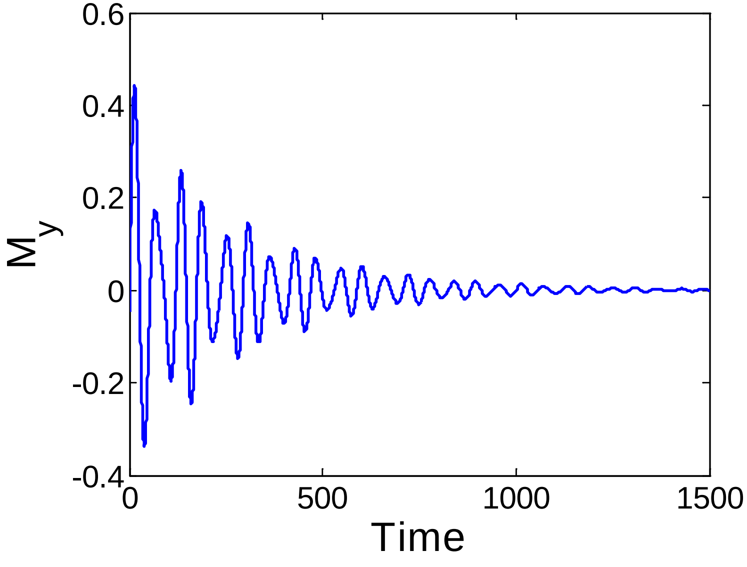



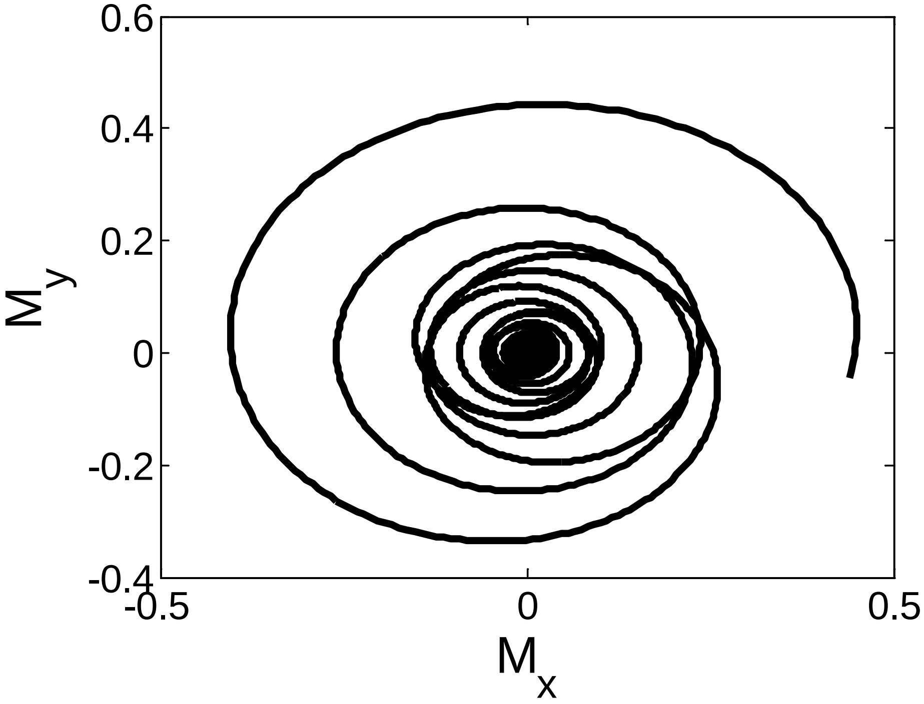
# Fourier transform of $P_x(t)$



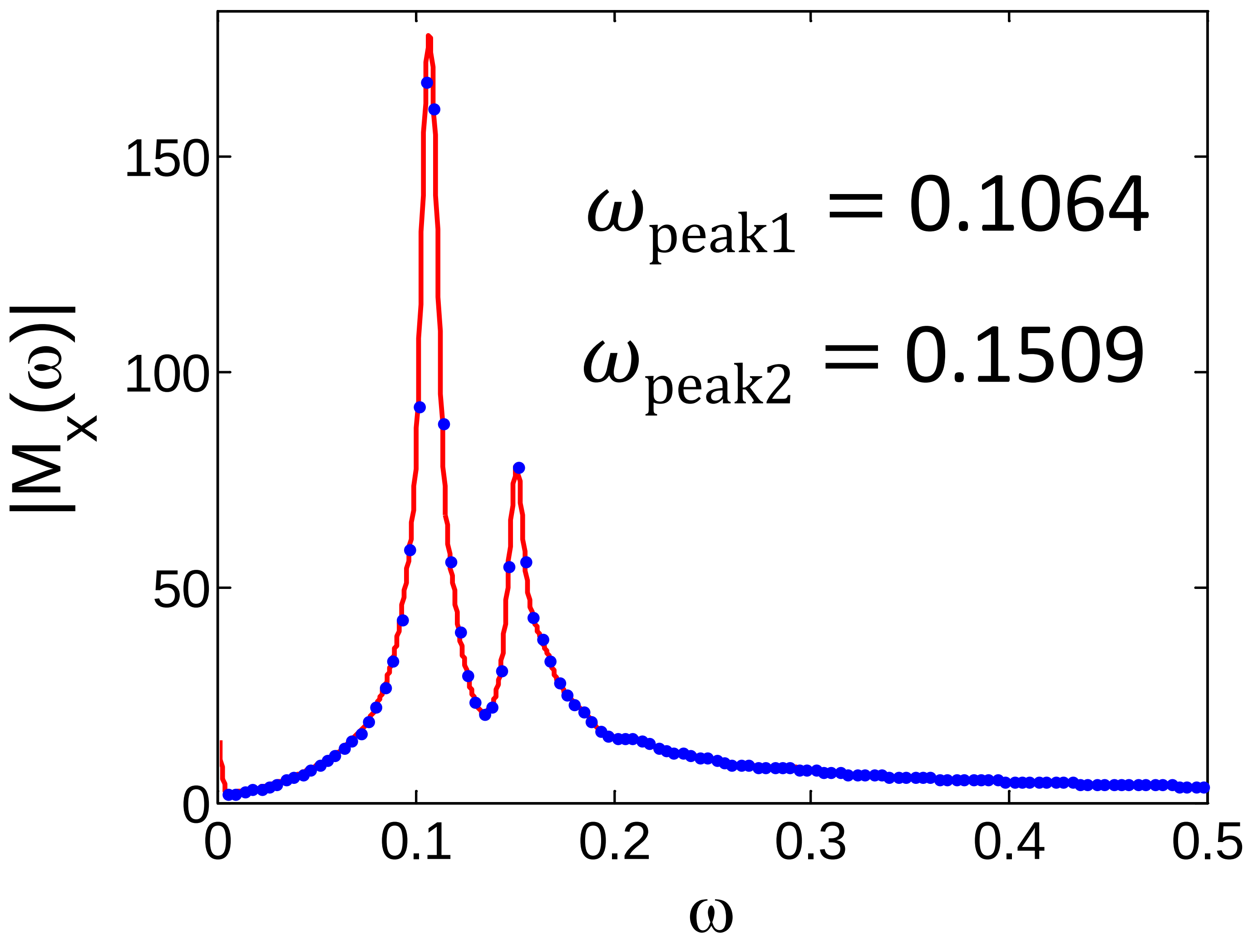
# Magnetization relaxation



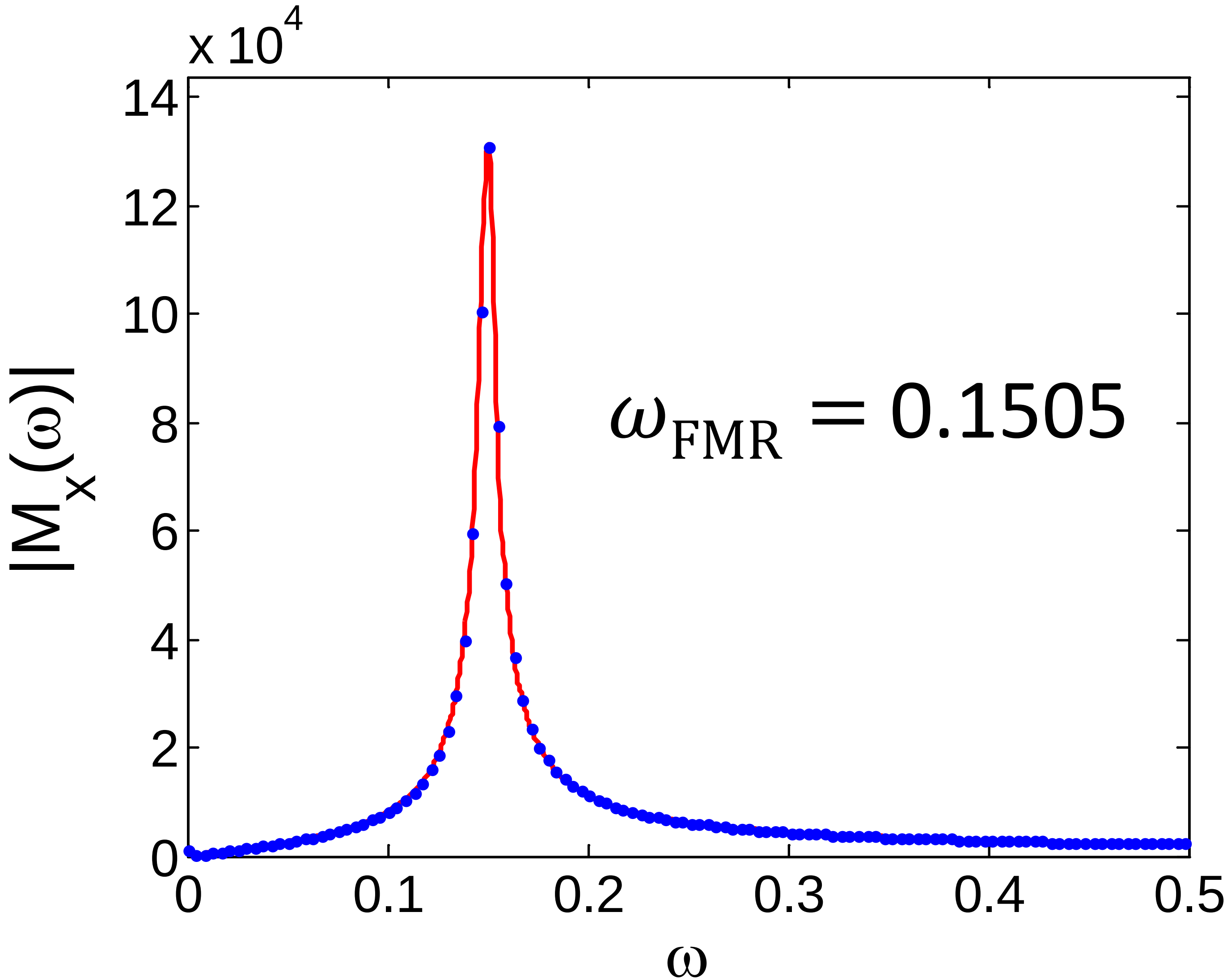



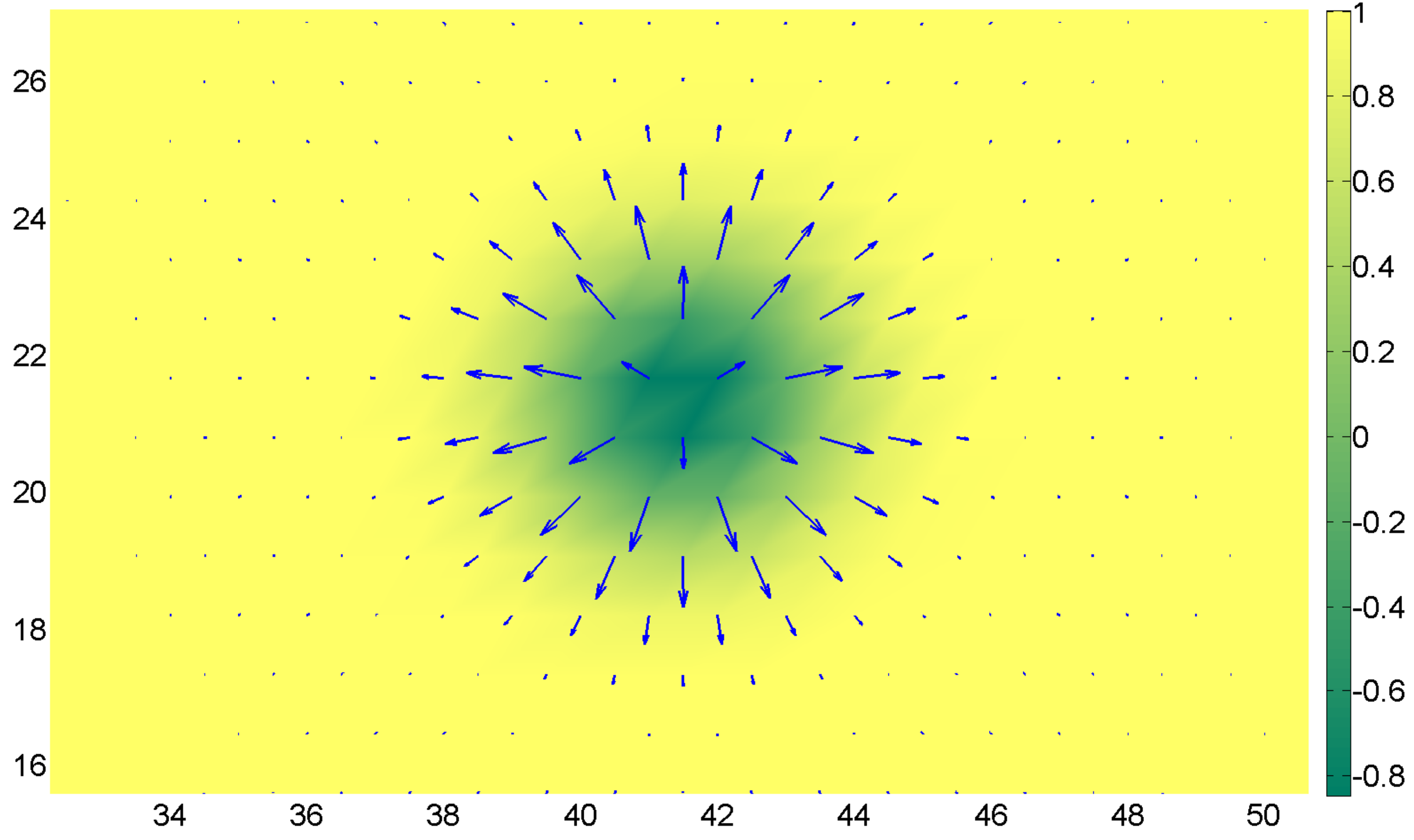
# Fourier transform of $M_x(t)$



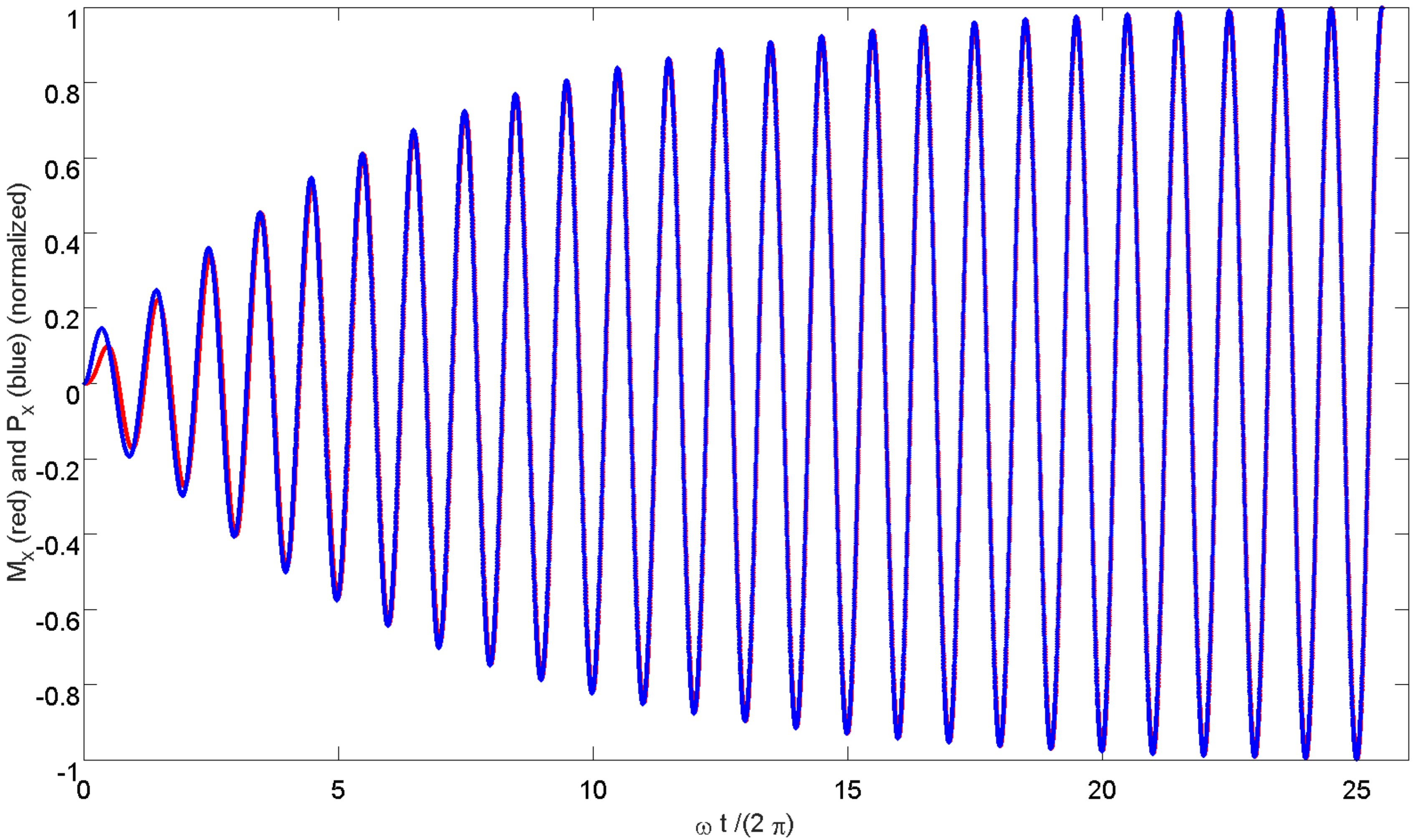
# Bulk ferromagnetic resonance



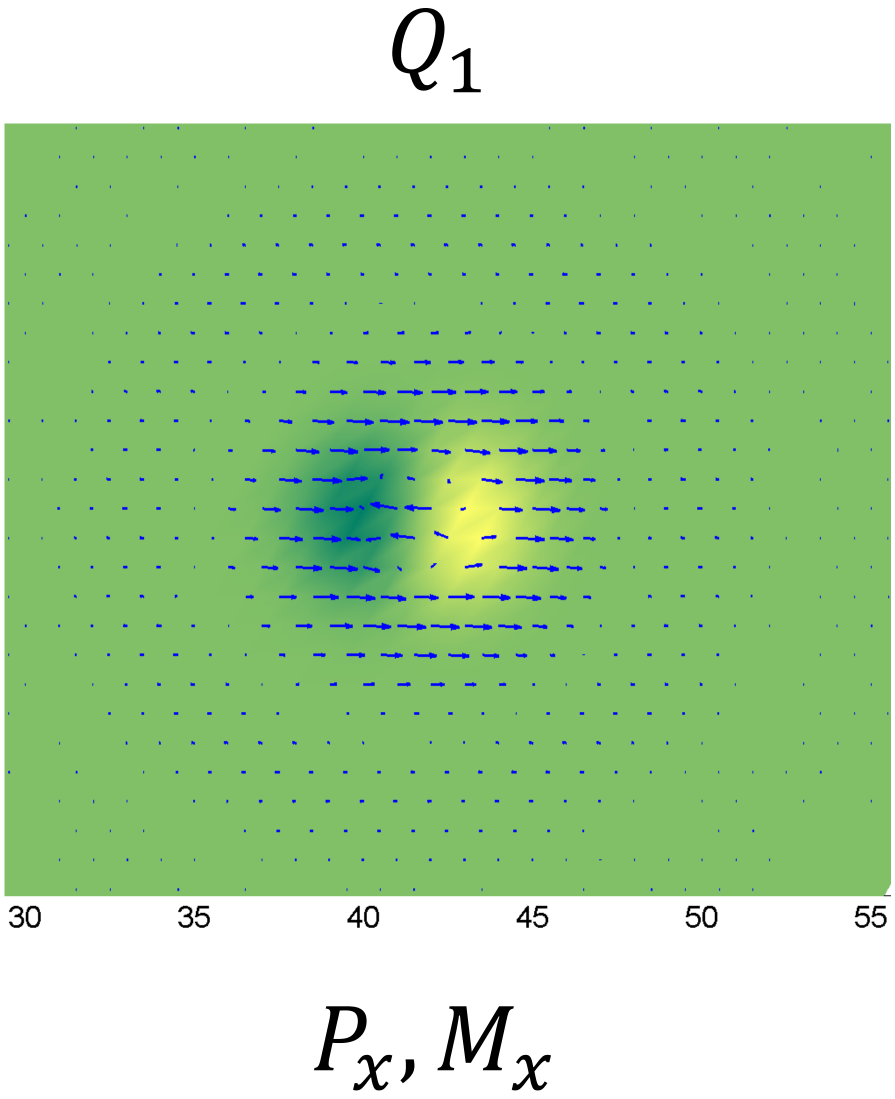
# Néel Skyrmion

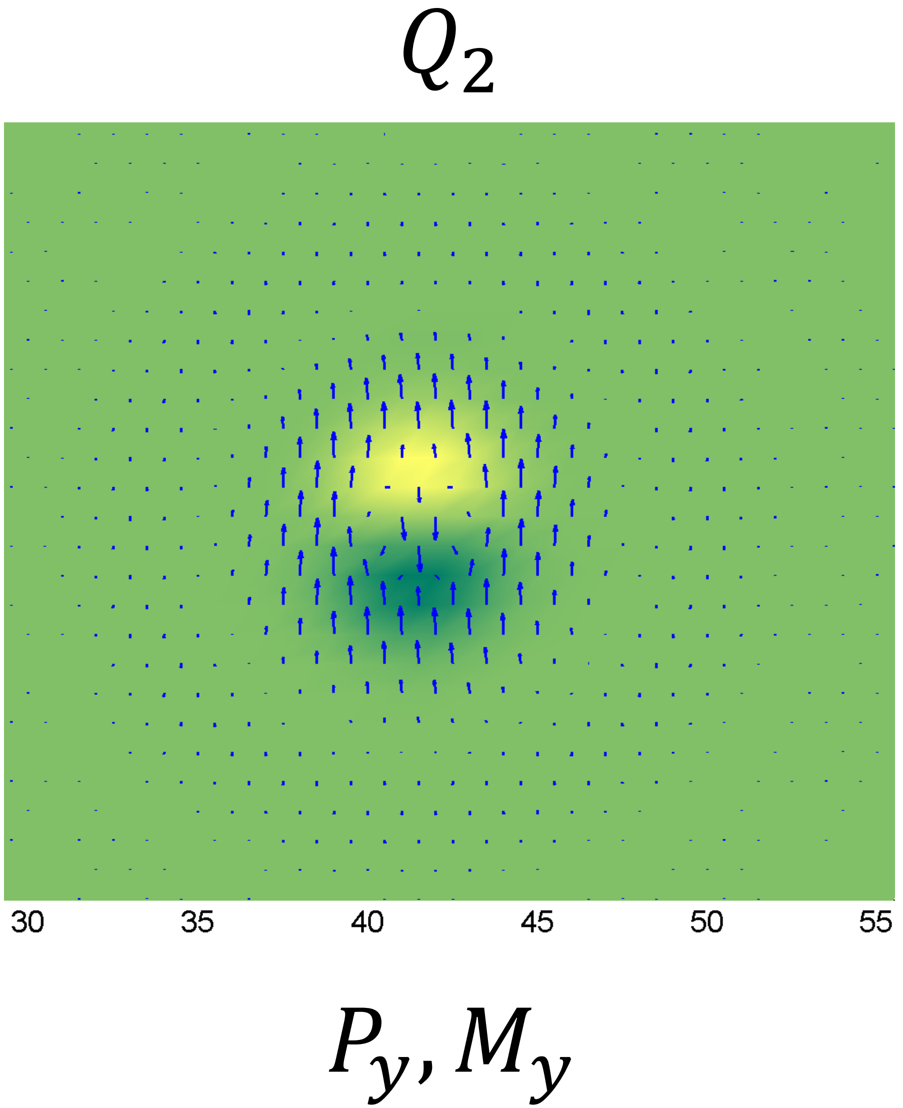


# Excitation of magnetic moment with electric field



# Localized mode with $\omega = 0.1068$



# Coupling to one mode

$$\frac{\left(1+\alpha^2\right)}{\omega_\nu}\ddot{\mathrm{X}}+\alpha\Gamma_{\nu 2,\nu 2}\dot{\mathrm{X}}+\dot{\mathrm{Y}}=0$$

$$\frac{\left(1+\alpha^2\right)}{\omega_\nu}\ddot{\mathrm{Y}}+\alpha\Gamma_{\nu 1,\nu 1}\dot{\mathrm{Y}}-\dot{\mathrm{X}}=0$$

$$\Gamma_{\nu 1,\nu 1}=\Gamma_{\nu 2,\nu 2}\approx 1.23$$

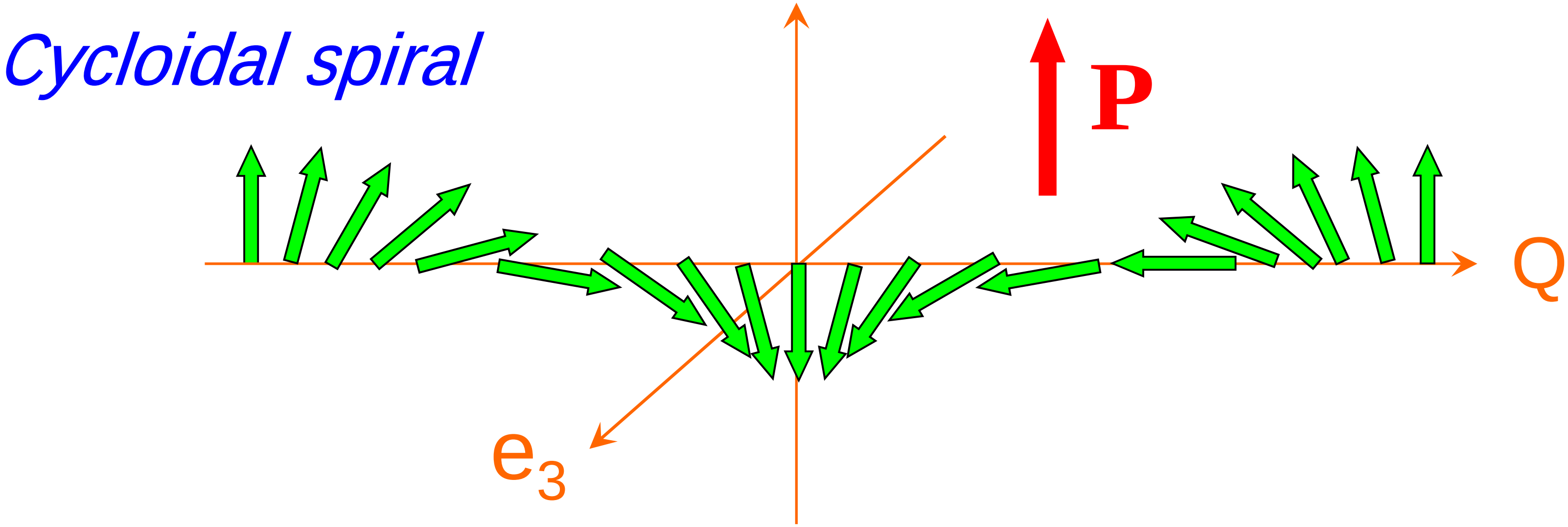
**which has the same form as**

$$\begin{cases} M\ddot{\mathrm{X}}+\alpha\Gamma\dot{\mathrm{X}}-G\dot{\mathrm{Y}}=0 \\ M\ddot{\mathrm{Y}}+G\dot{\mathrm{X}}+\alpha\Gamma\dot{\mathrm{Y}}=0 \end{cases}$$

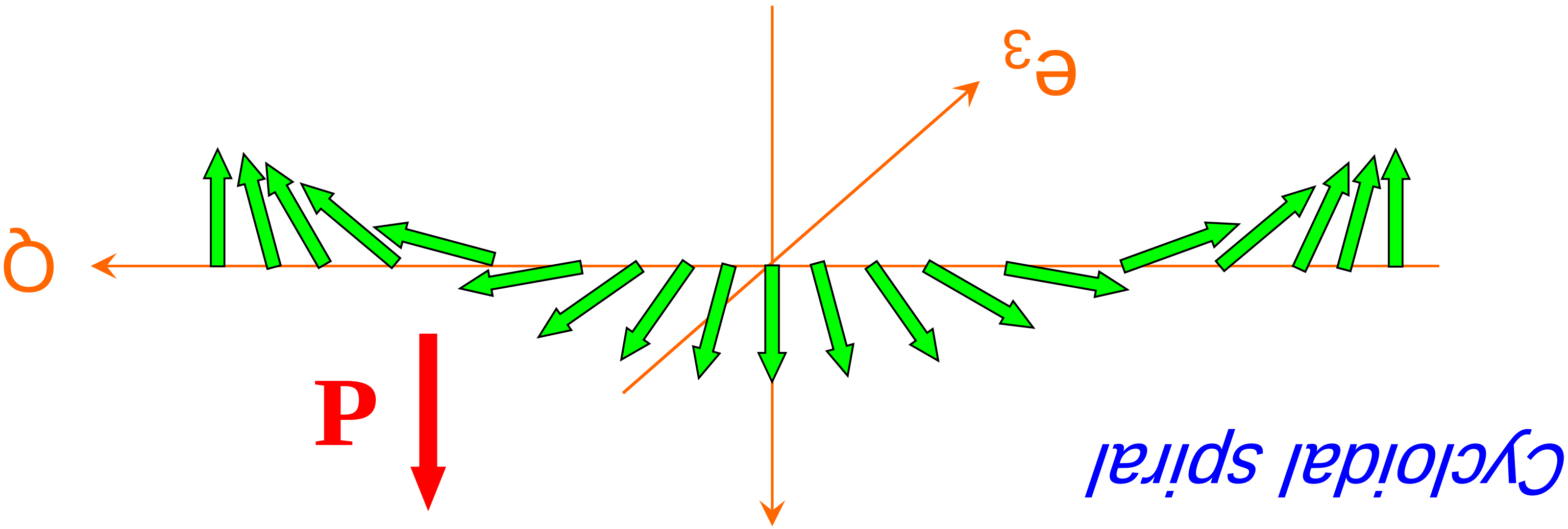
$$M=\left(1+\alpha^2\right)\frac{|G|}{\omega_\nu}$$

$$\frac{\Gamma}{|G|}\approx 1.25$$

# Breaking of inversion symmetry by spin ordering




Inversion I: $(x,y,z) \rightarrow (-x,-y,-z)$



# Skyrmion electric dipole moment

**Magneto-electric coupling**

$$f_{me} = -g\,\mathbf{E}\cdot\left[\mathbf{m}(\nabla\cdot\mathbf{m}) - (\mathbf{m}\cdot\nabla)\mathbf{m}\right]$$

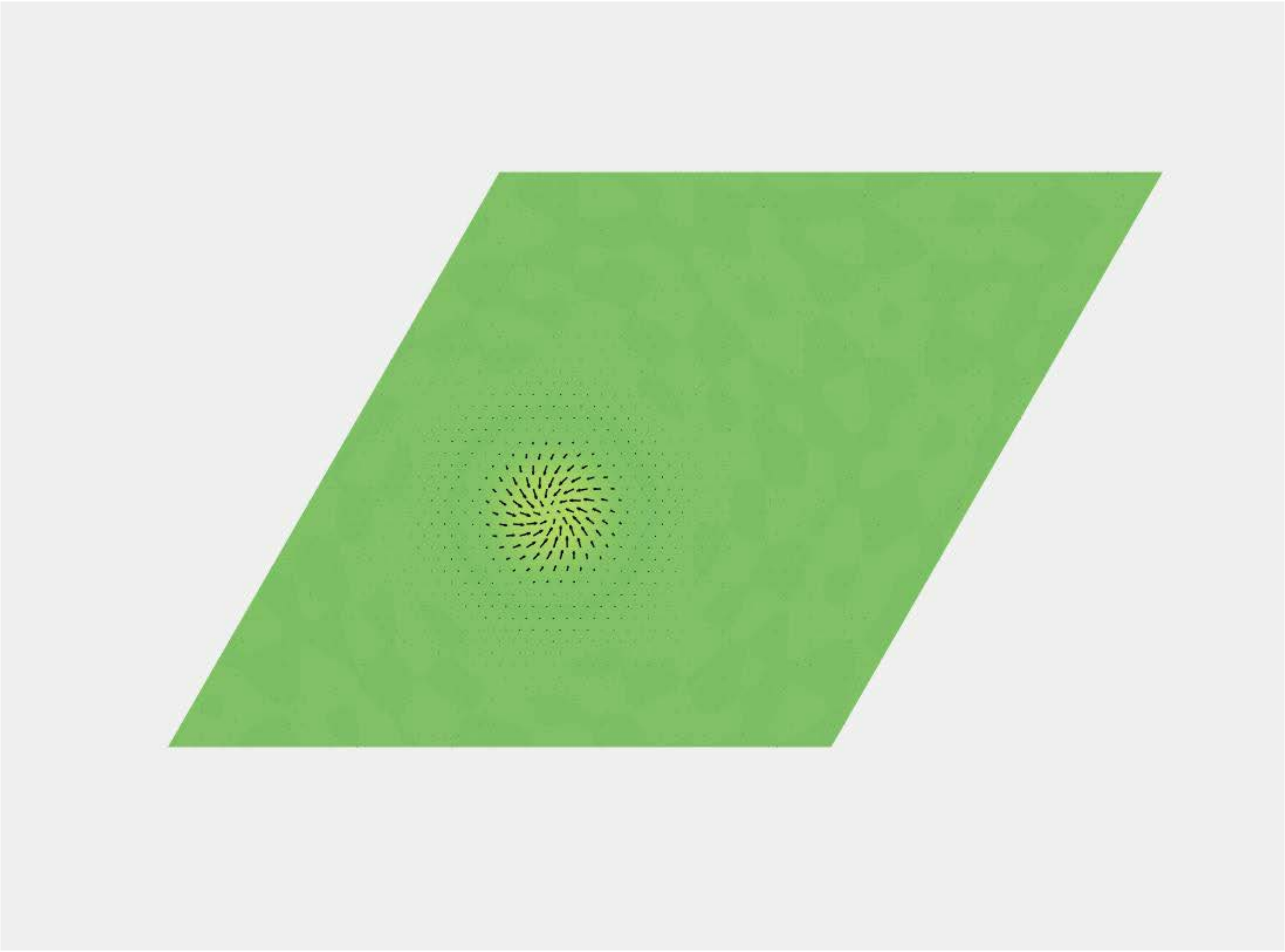
**Polarization induced by skyrmion**

$$P_z = -\frac{\partial f}{\partial E_z} = g\left[\frac{d\Theta}{dr} + \mathrm{v}\frac{\sin 2\Theta}{2r}\right]\boxed{\cos\left((\mathrm{v}-1)\varphi + \chi\right)}$$

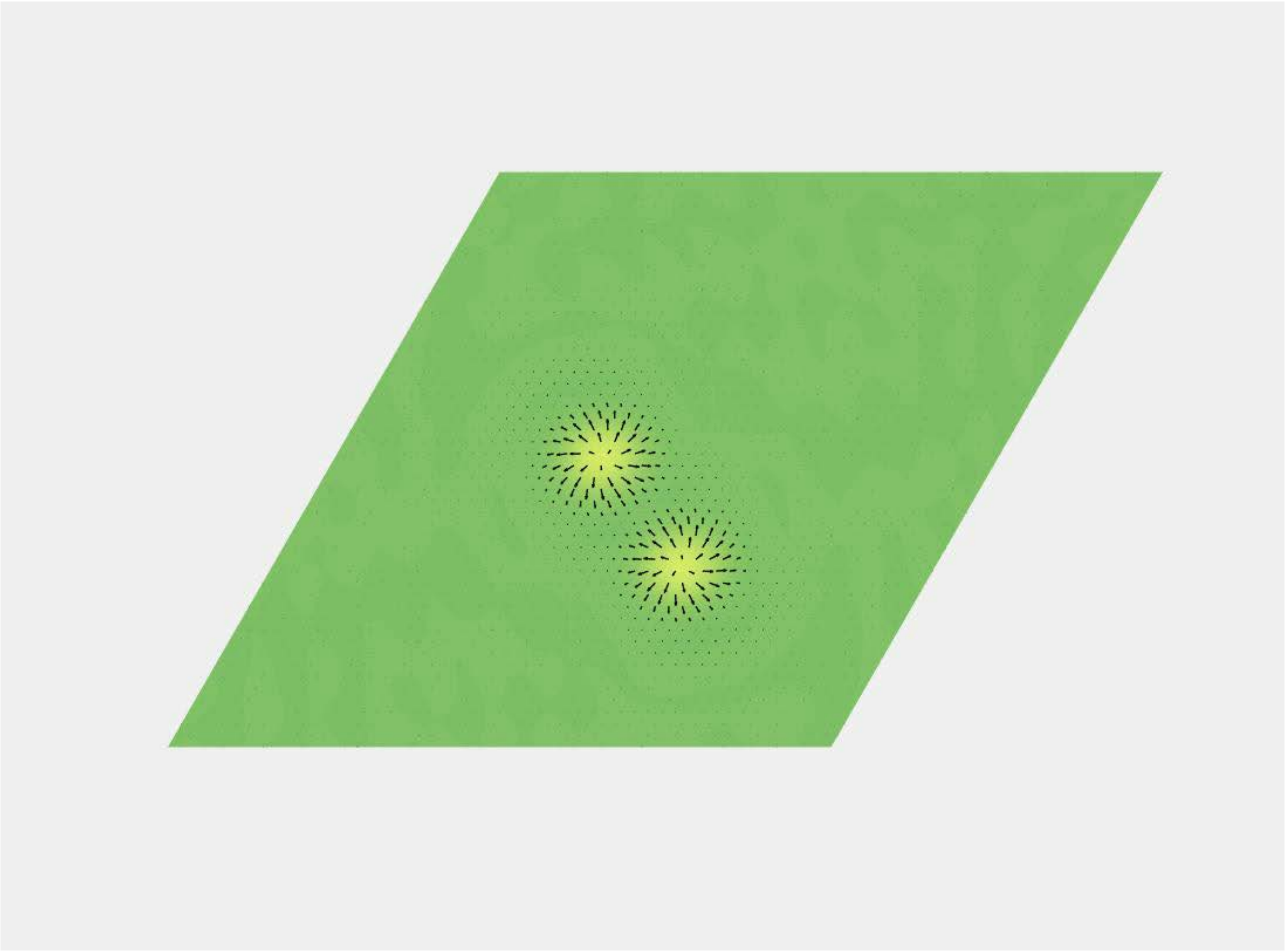
**Dipole moment for vorticity v = +1**

$$D_z(\chi) = D(0)\boxed{\cos\chi}$$

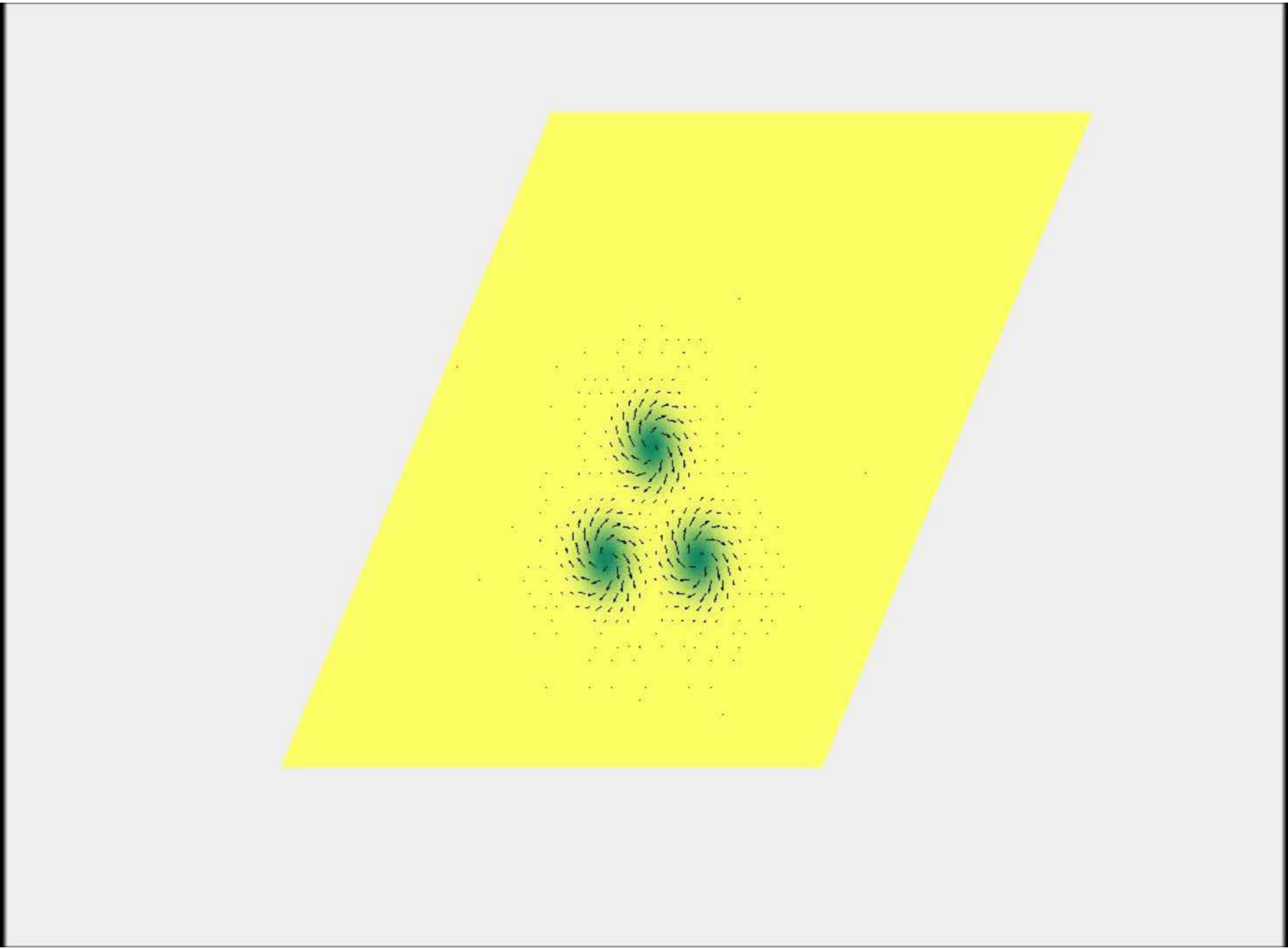
# Electric excitation of skyrmion



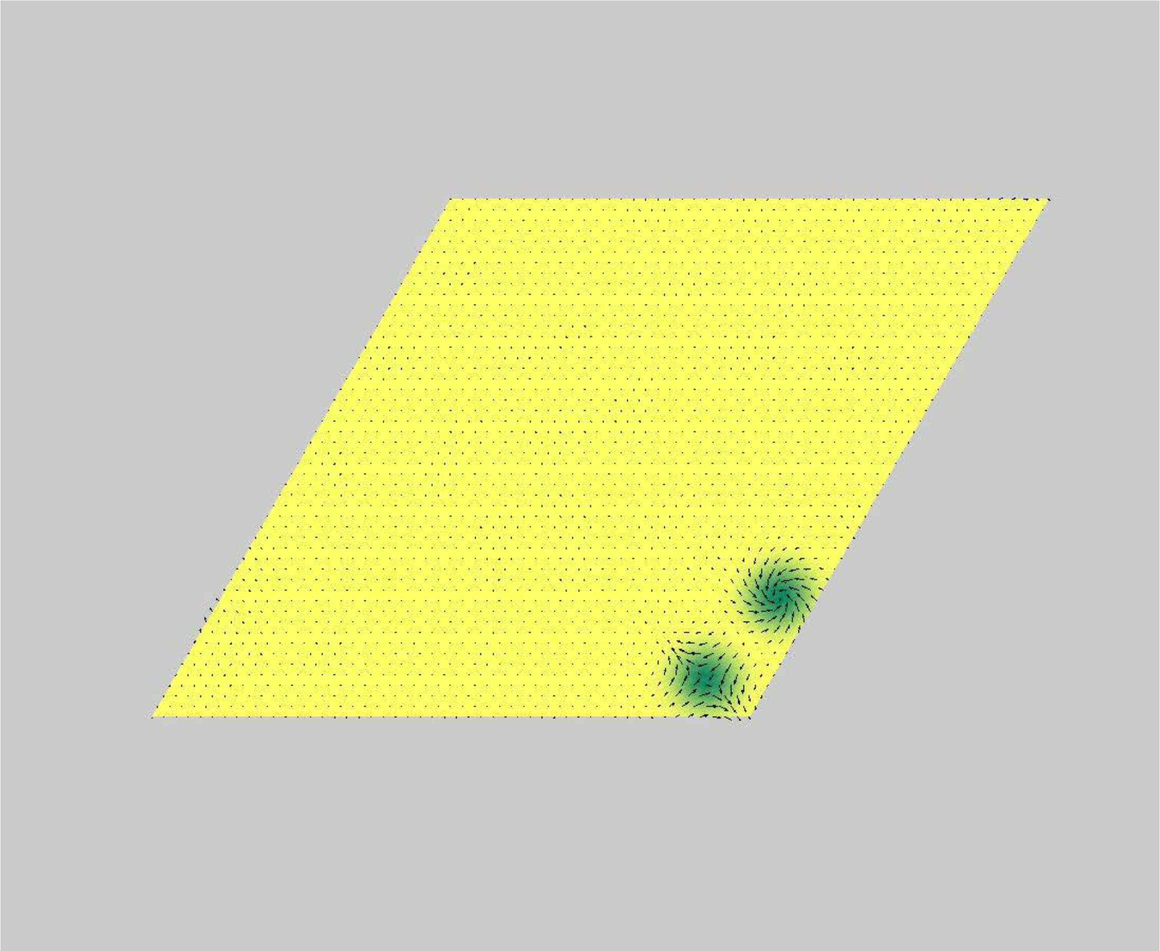
# $E_z$-induced rotation of skyrmion-skyrmion pair



# $E_z$-excitation of 3 skyrmions



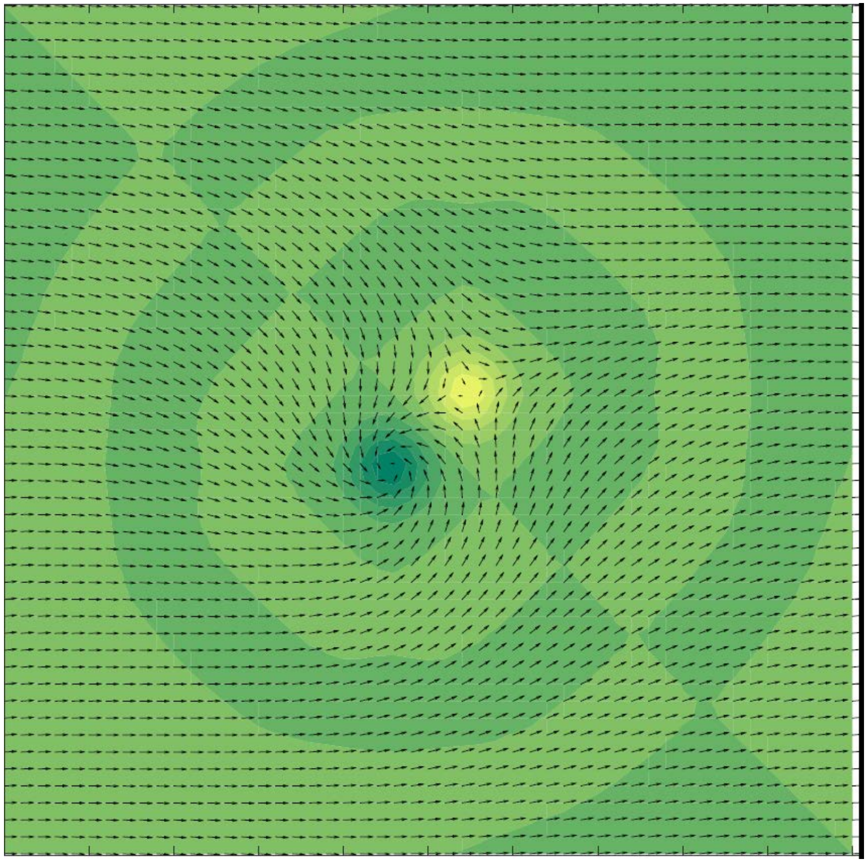
# $E_z$-induced motion of skyrmion-antiskyrmion pair



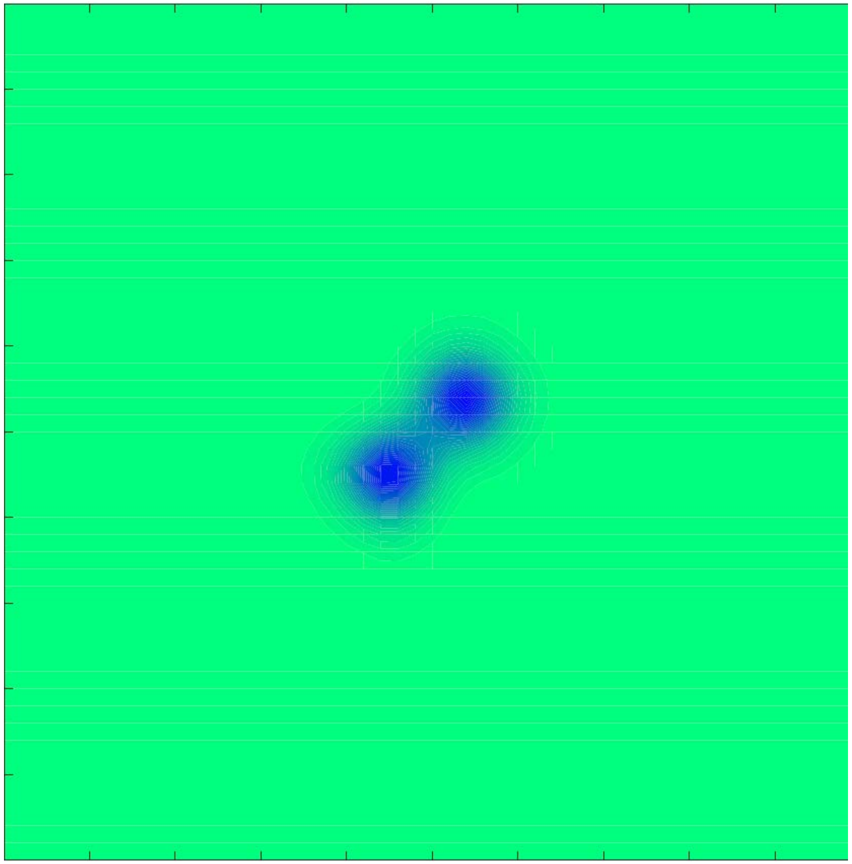
# Bi-merons

## easy plane anisotropy

**Spin configuration**



**Topological density**



*Y. Kharkov, O. P. Sushkov & MM, PRL 2017*

# Electric charge of magnetic vortex

*MM PRL **96**, 067601 (2006)*

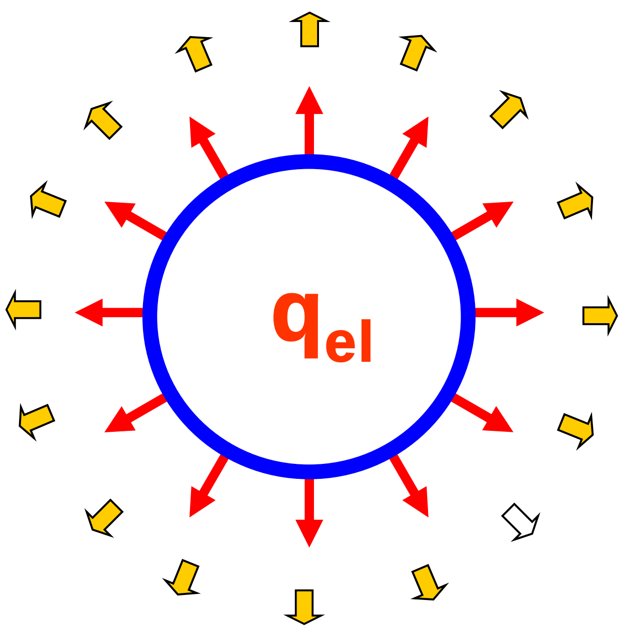
**ME coupling**

$$f_{me} = -g\,\mathbf{E}\cdot\left[\mathbf{m}(\nabla\cdot\mathbf{m}) - (\mathbf{m}\cdot\nabla)\mathbf{m}\right]$$

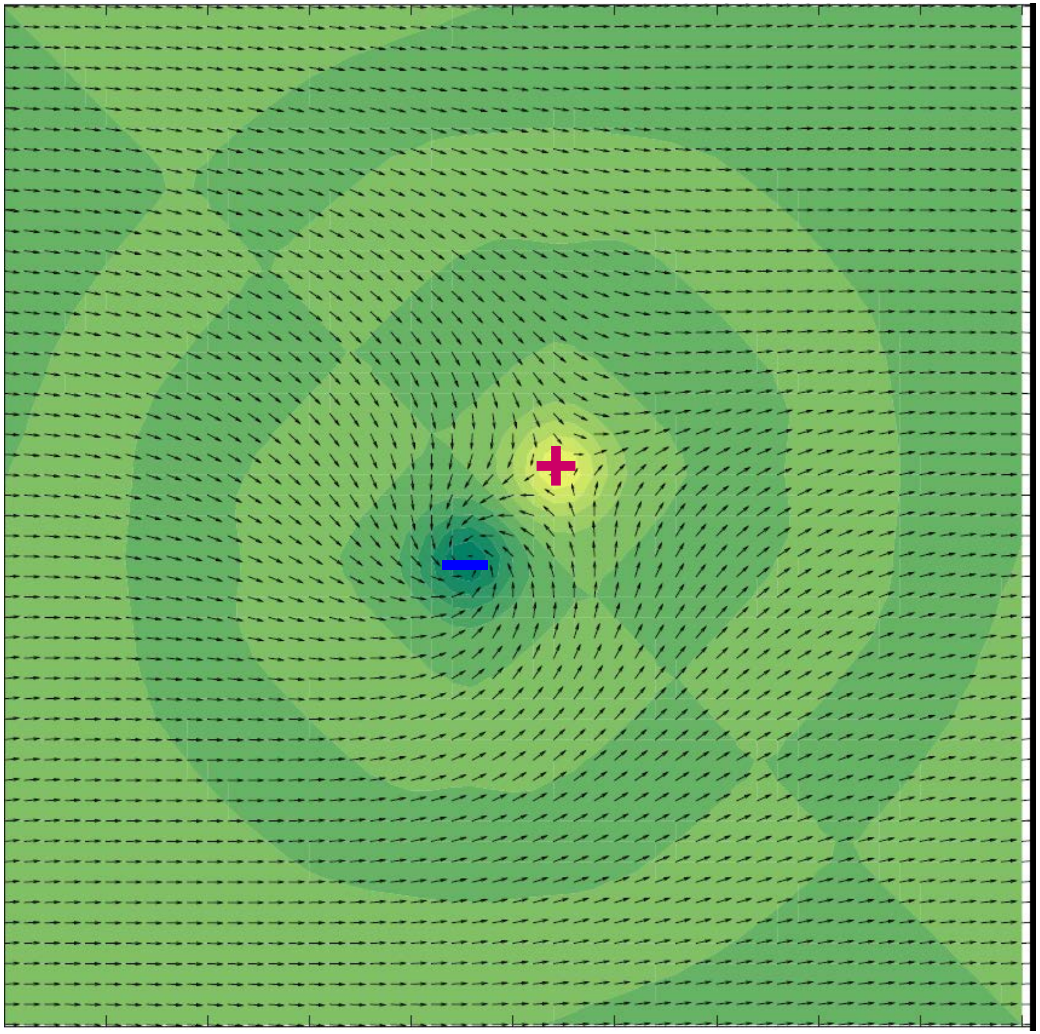
**Force**

$$\mathbf{F} = -\frac{\partial f_{me}}{\partial \mathbf{X}} = -g\mathbf{E}\oint_{\partial G} dx_i\left(\hat{\mathbf{z}}\cdot\mathbf{m}\times\partial_i\mathbf{m}\right)$$

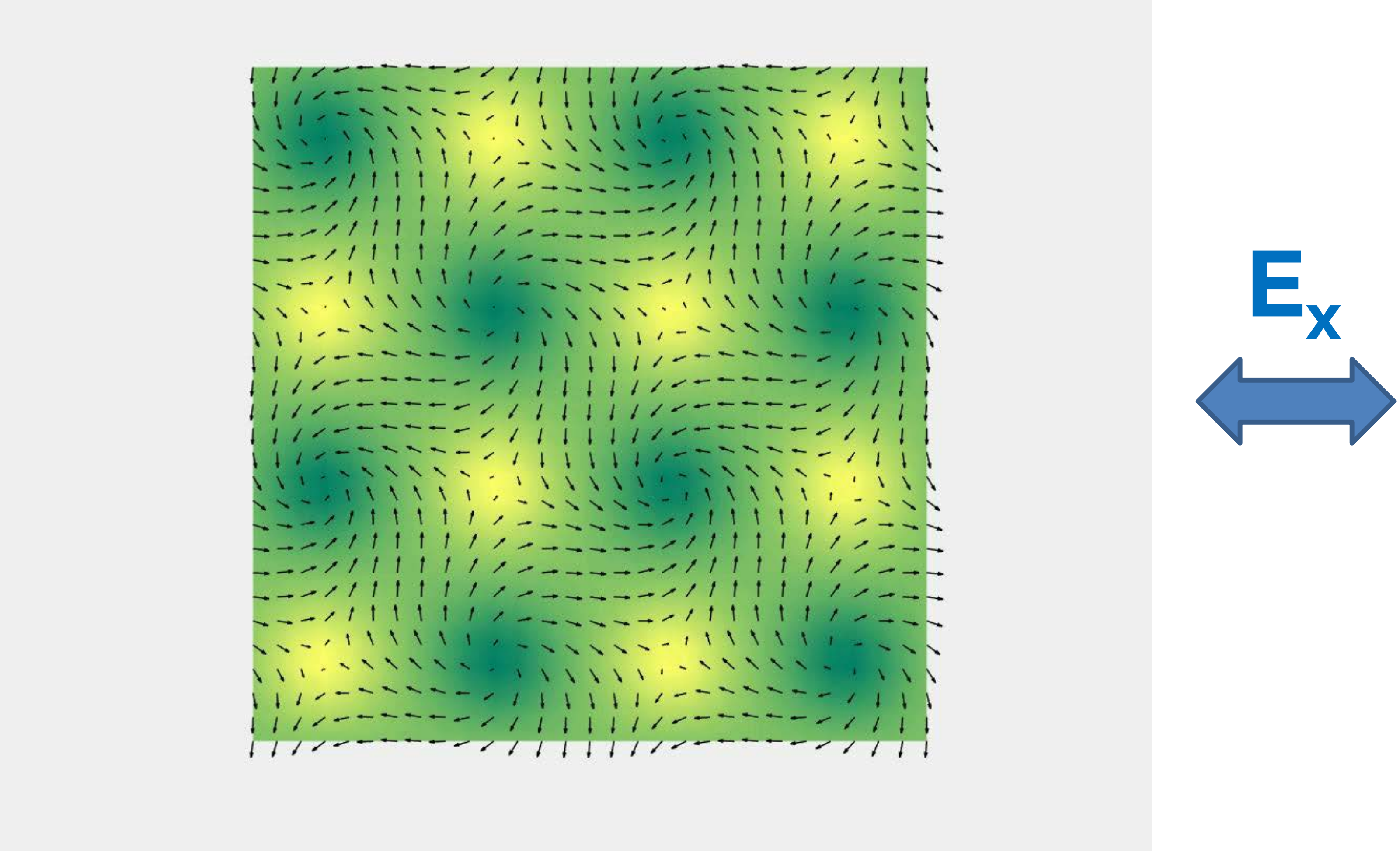
**winding number**



# Electrically-charged merons



# $E_x$ excites plasma oscillations along y

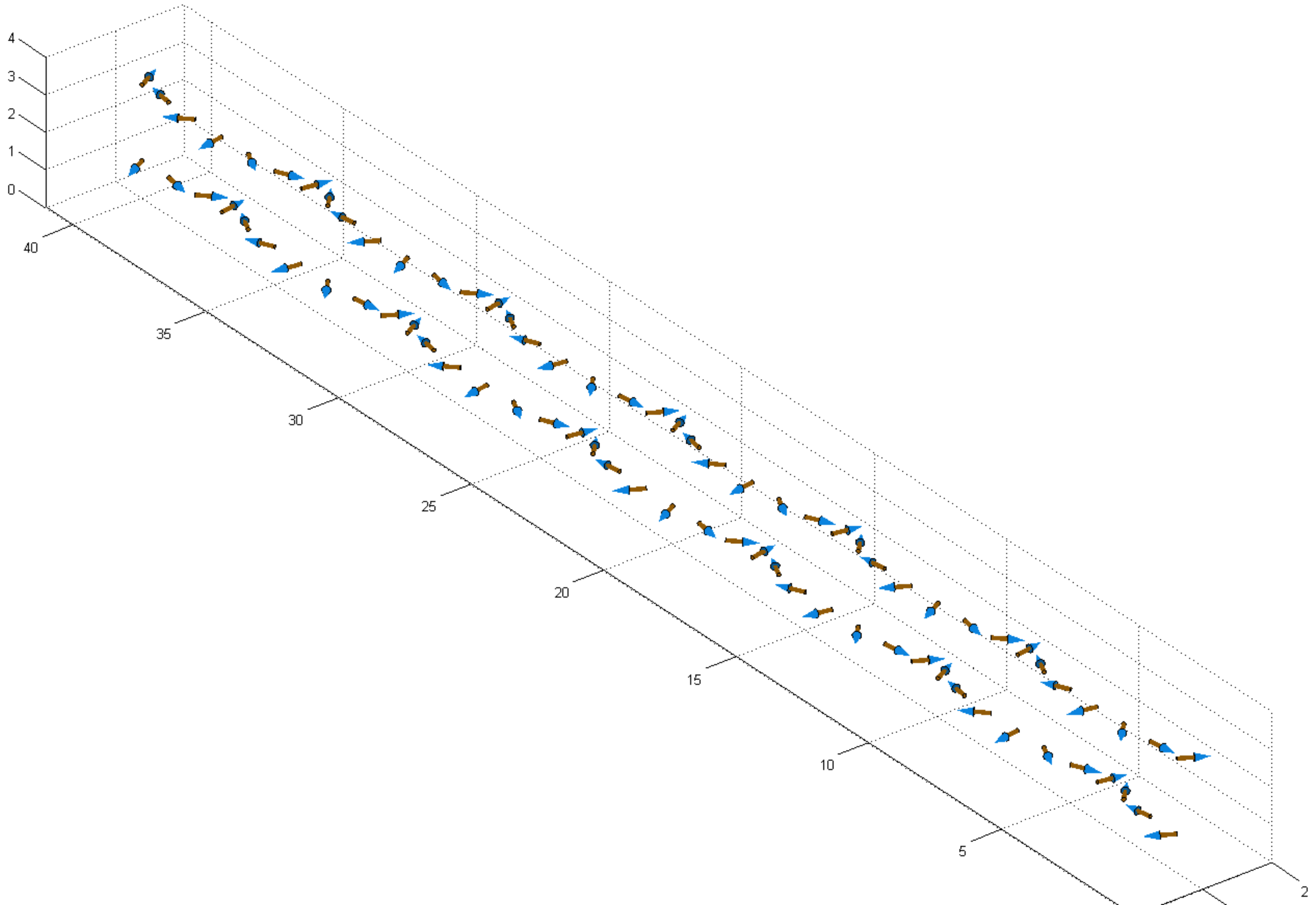


# Summary

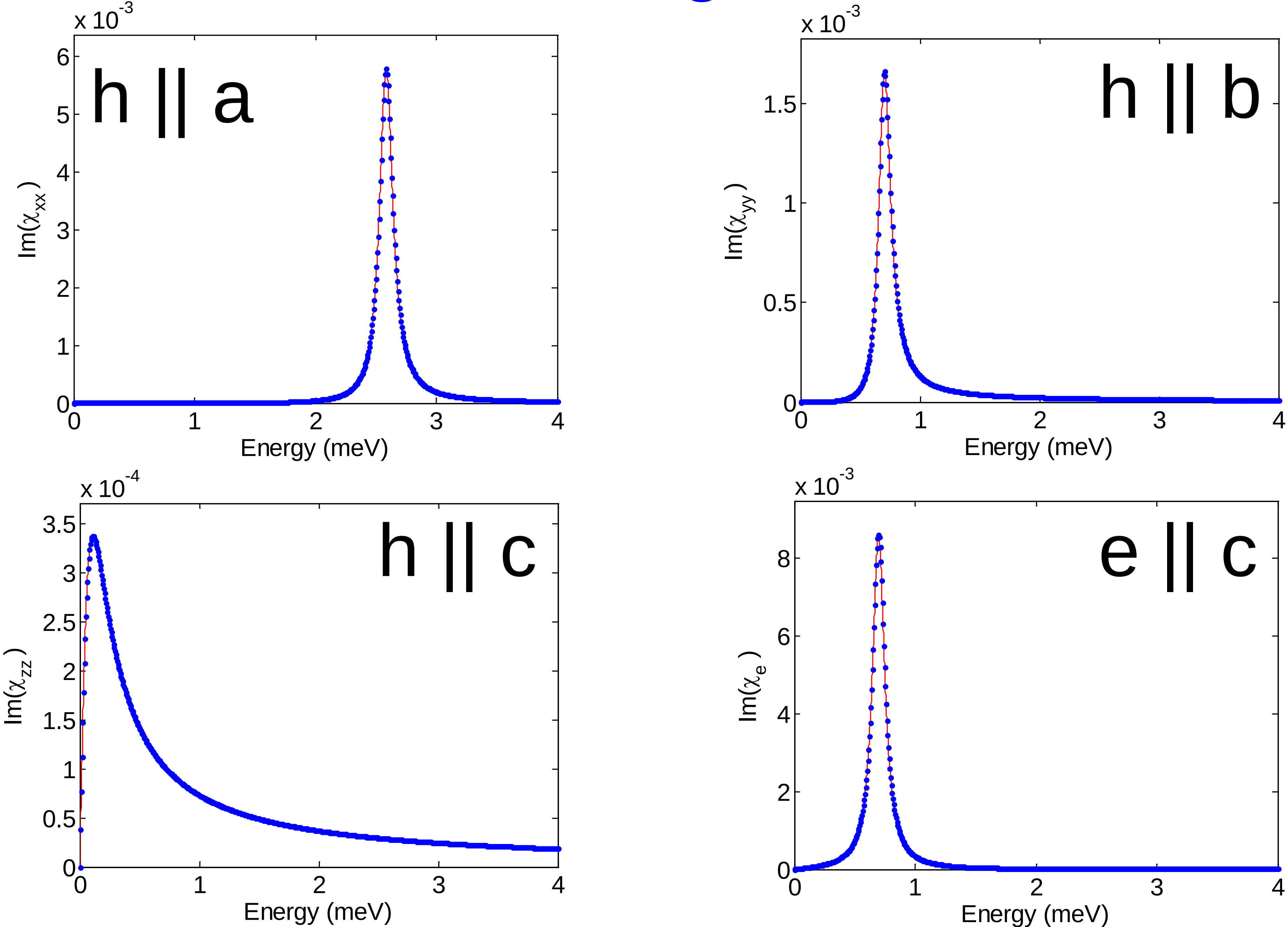
- Magnetoelectric coupling allows for electric field control of light propagation through non-collinear spin textures and excitation of dynamics of topological magnetic defects with an oscillating electric field.

- Low energy losses in Mott insulators.

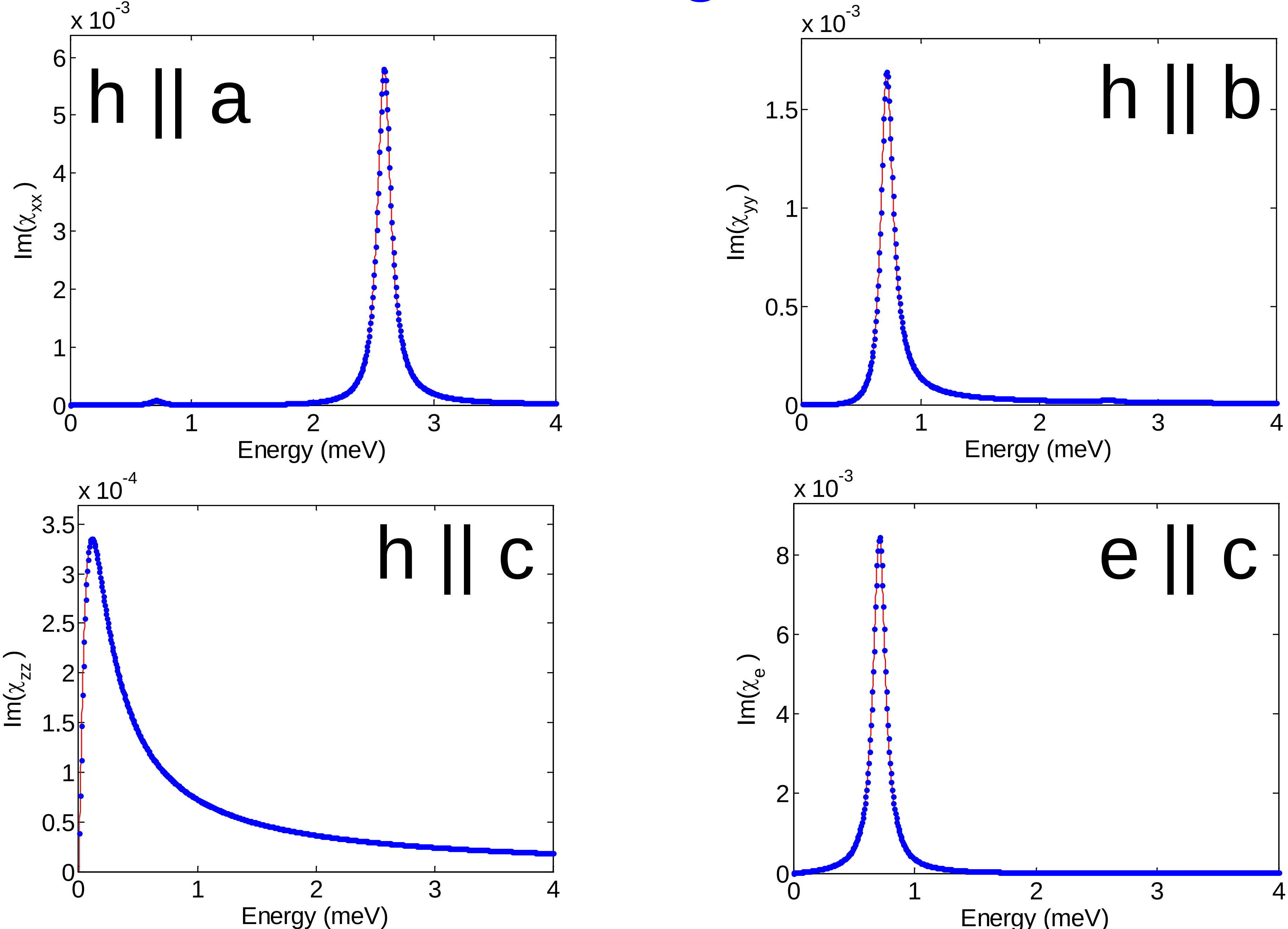
# Spin configuration



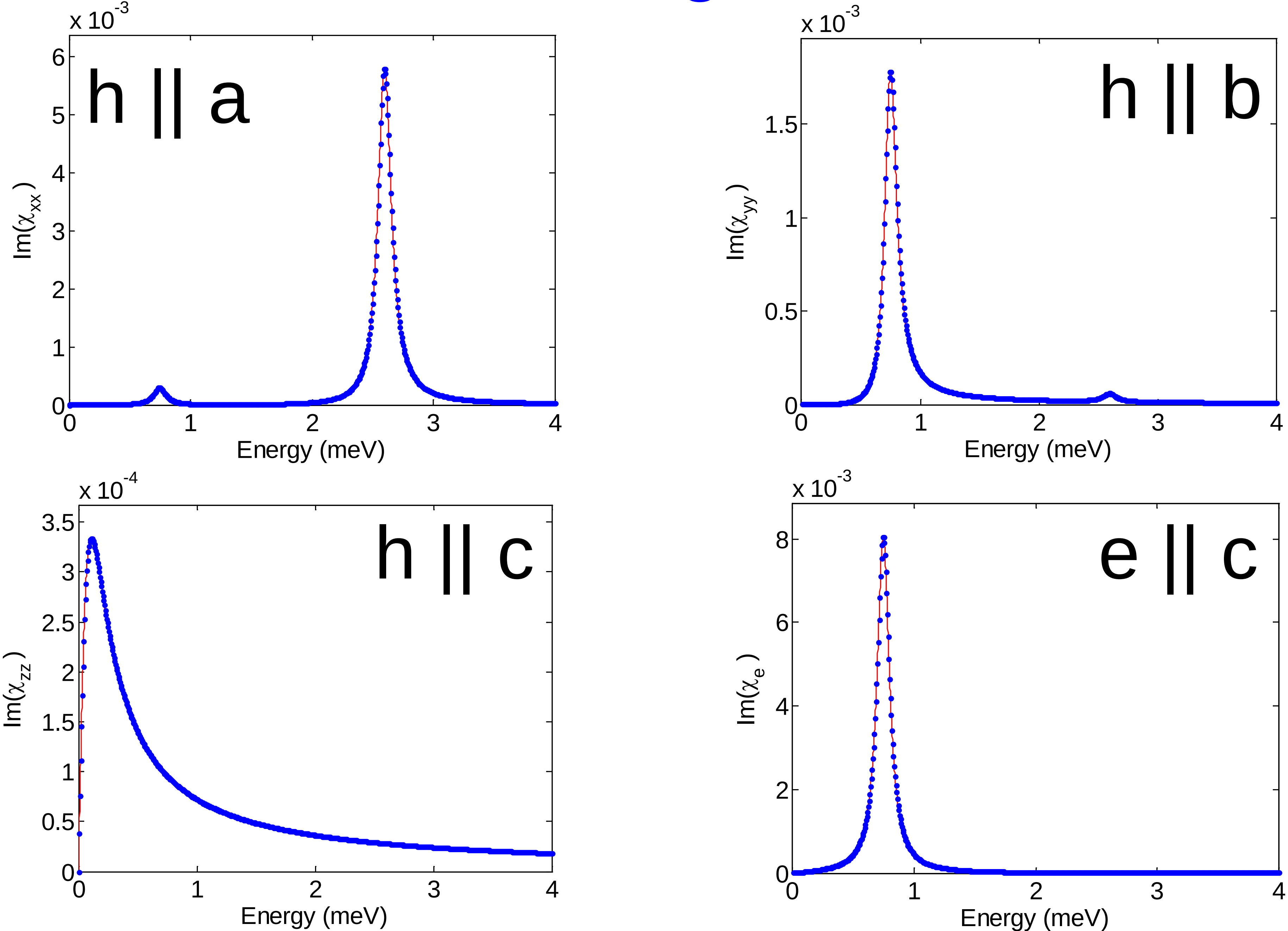
# AFMR and E-magnons at H = 0T



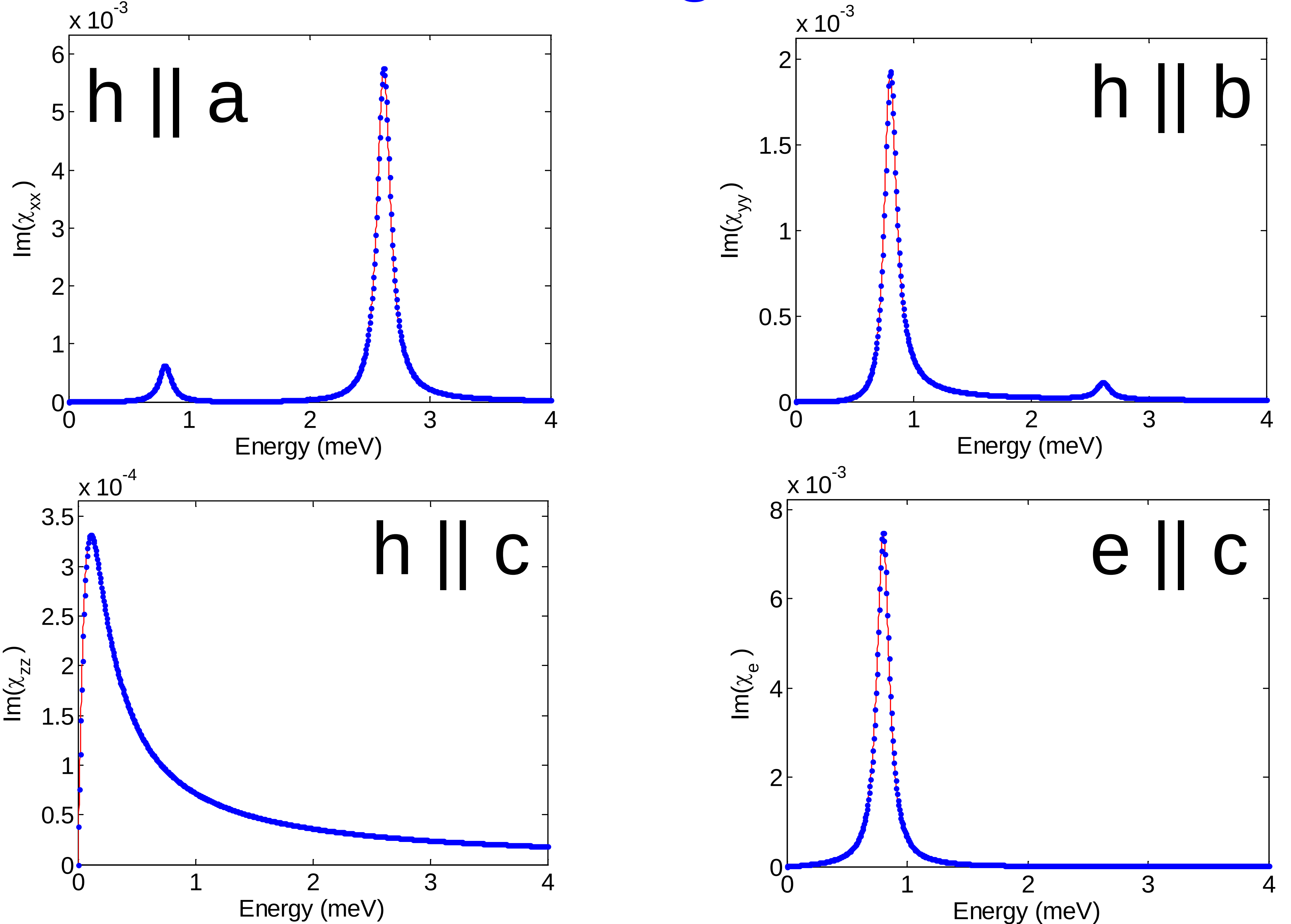
# AFMR and E-magnons at H = 1T



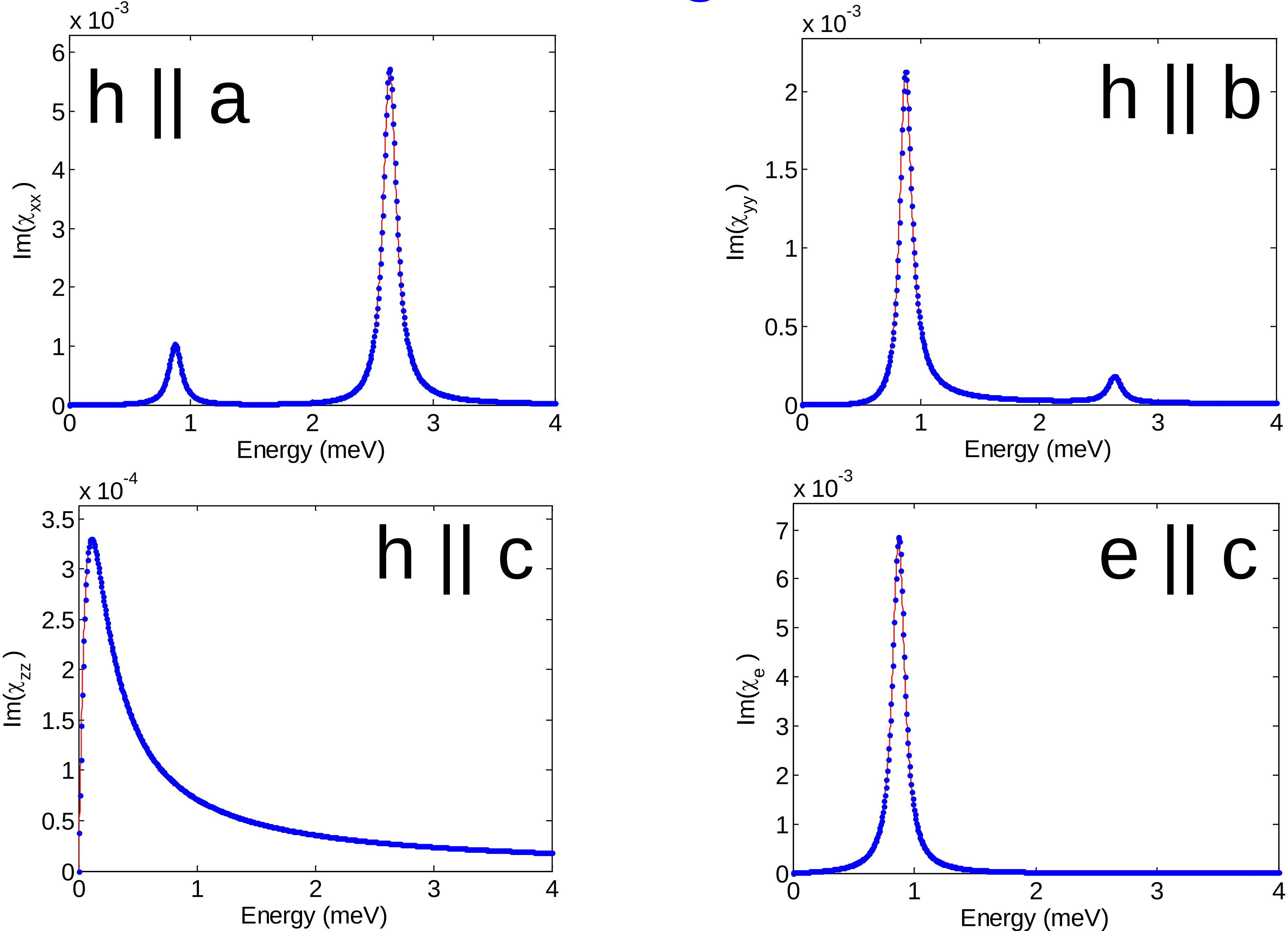
# AFMR and E-magnons at H = 2T



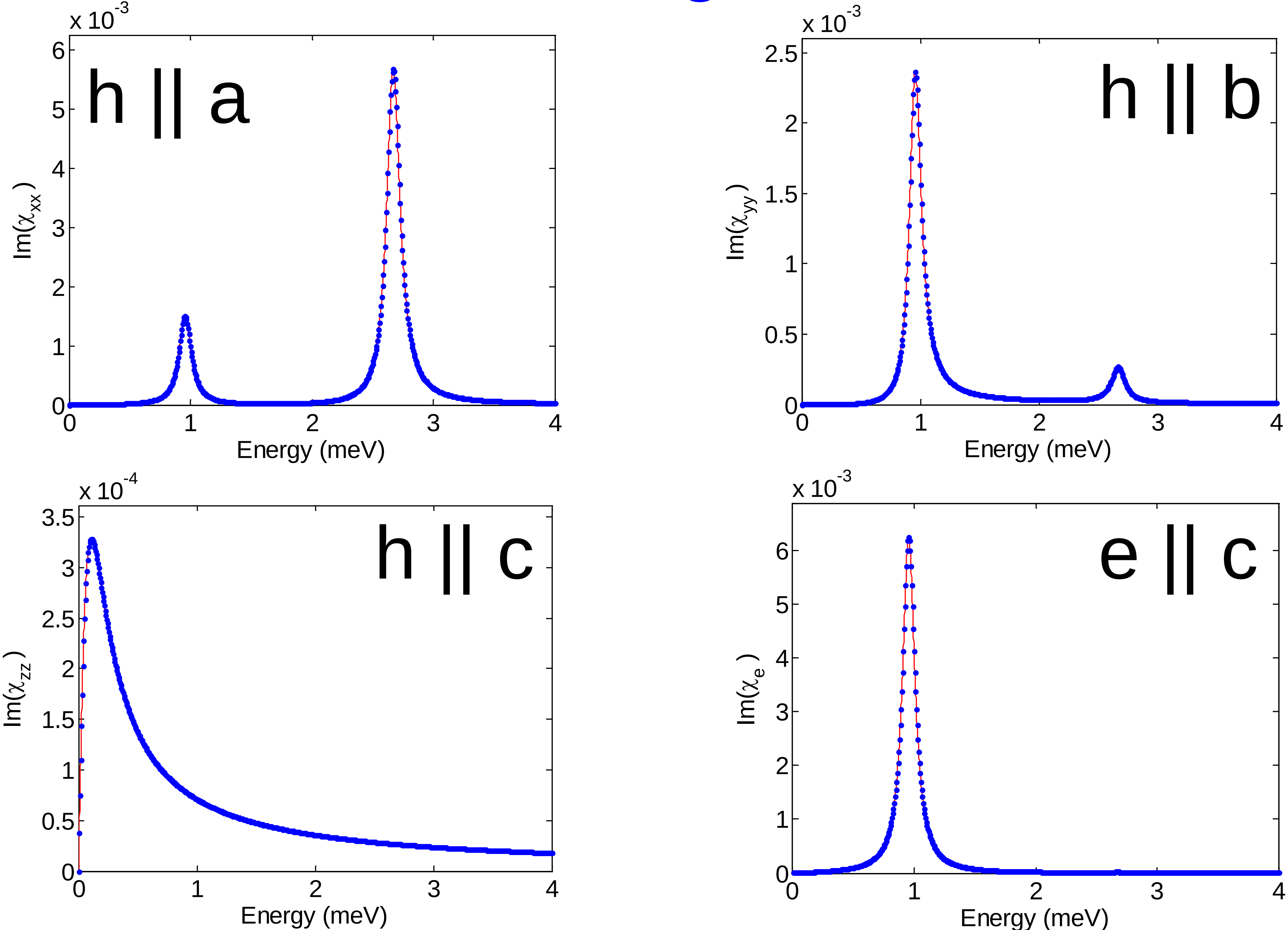
# AFMR and E-magnons at H = 3T



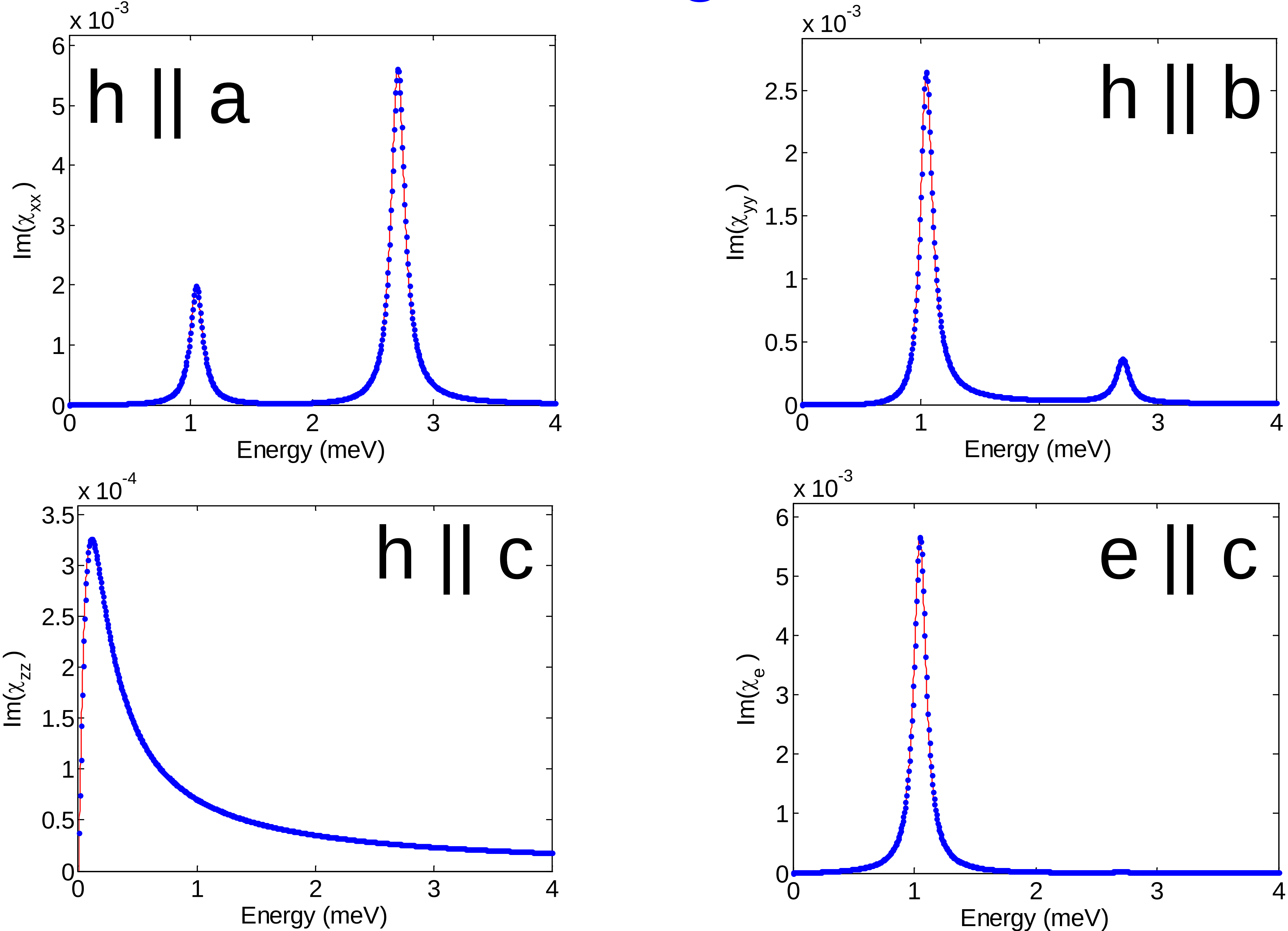
# AFMR and E-magnons at H = 4T



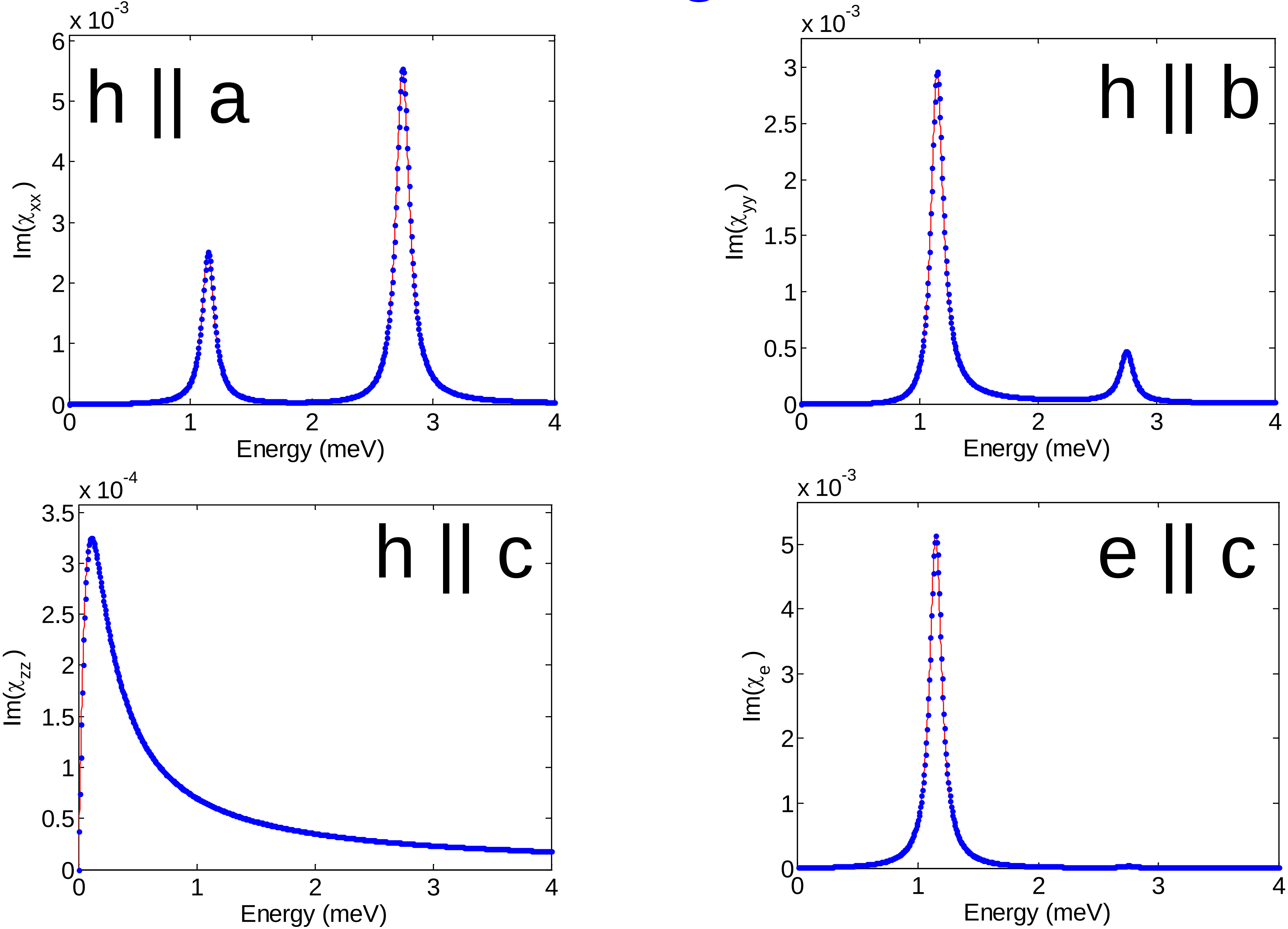
# AFMR and E-magnons at H = 5T



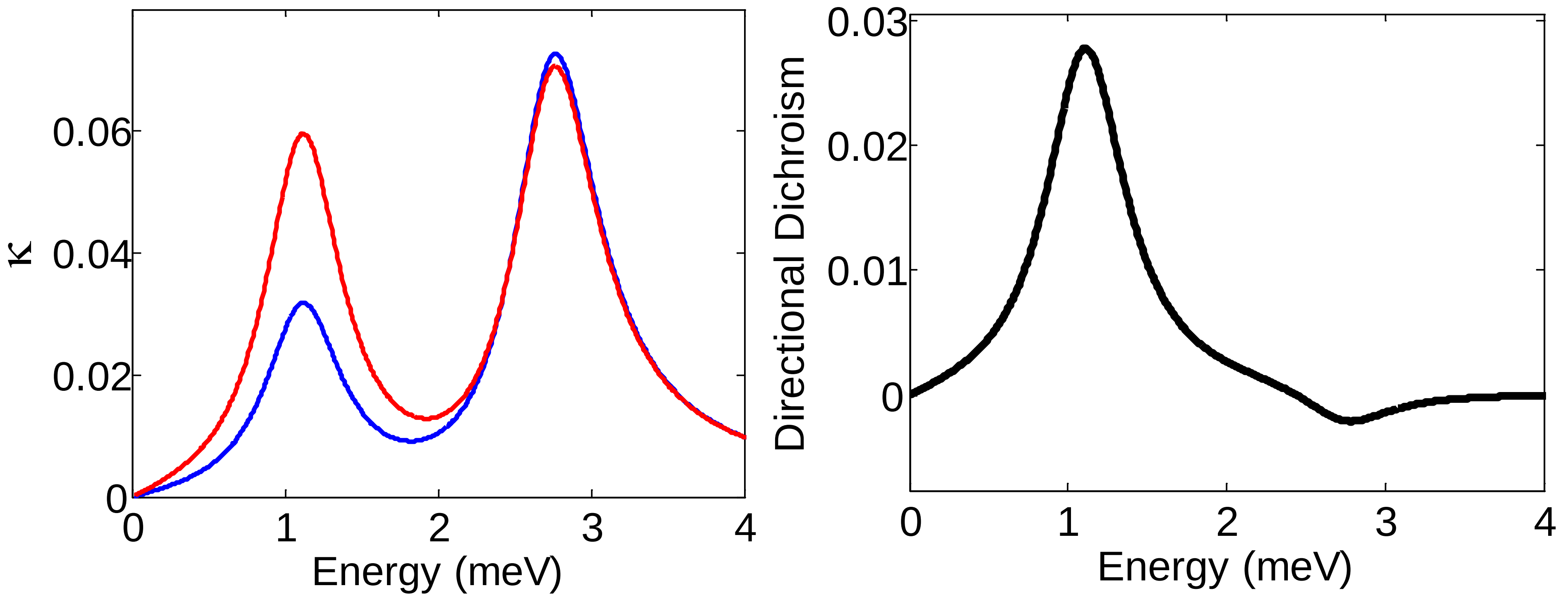
# AFMR and E-magnons at H = 6T



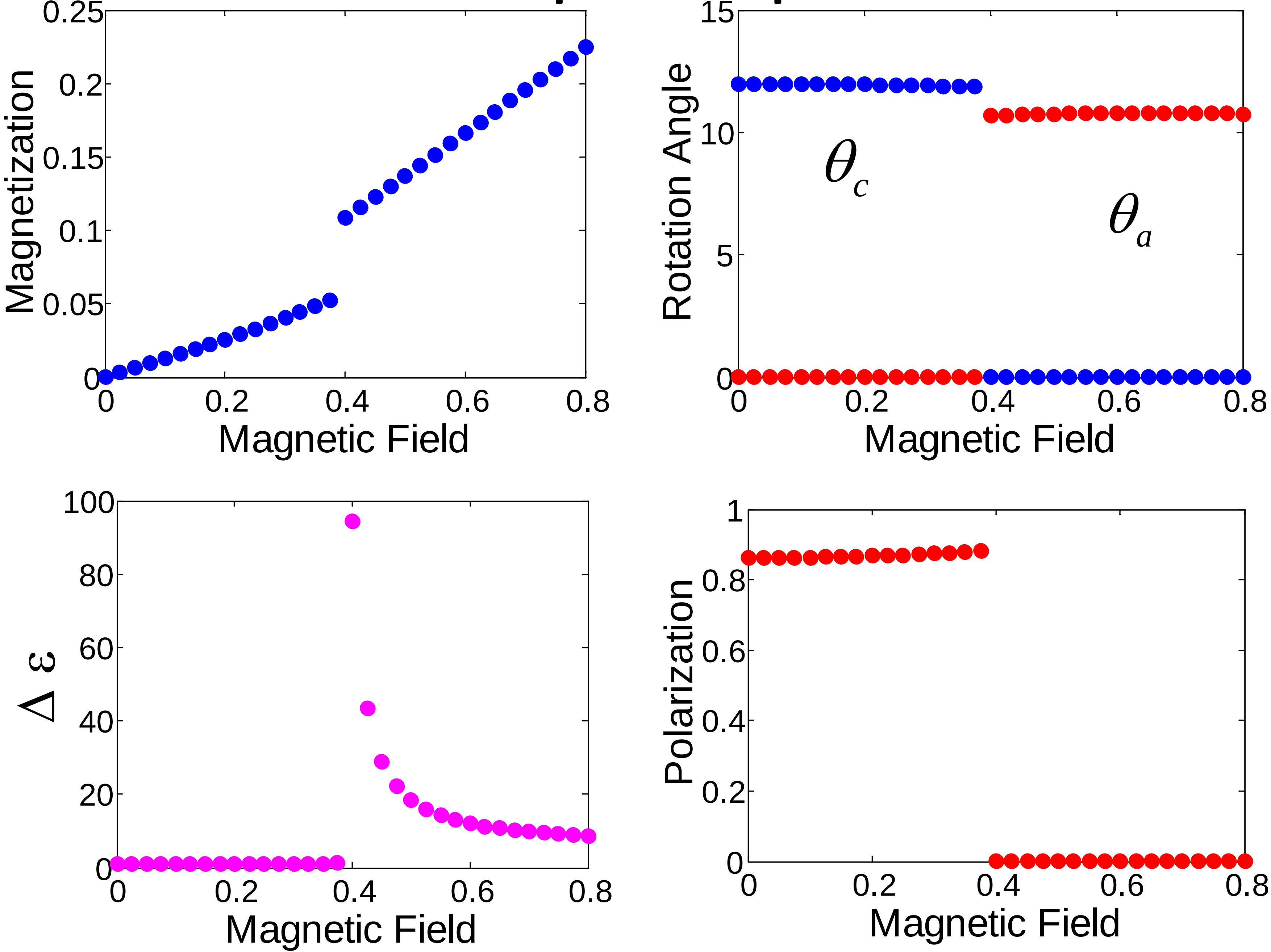
# AFMR and E-magnons at H = 7T



# Extinction coefficient and directional dichroism at H = 7T



# Spin flop



Magnetization
0.25
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
Magnetic Field
Rotation Angle
15
10
5
0
$\theta_c$
$\theta_a$
Magnetic Field
$\Delta\ \varepsilon$
100
80
60
40
20
0
Magnetic Field
Polarization
1
0.8
0.6
0.4
0.2
0
Magnetic Field